

# Housing Debt and Consumption\*

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June 2008

## Abstract

This paper analyses the effects of house price uncertainty and employment risk on consumption and savings in a finite horizon life-cycle model. The individual has a portfolio composed of two financial assets: a mortgage and a riskless asset in which she can borrow and save. In each period the consumer decides whether to withdraw equity from the house or not, subject to a transaction cost and a constraint on the amount that can be borrowed (up to 100% mortgages are possible). In a CARA framework, we derive a closed-form solution for the value function, the optimal refinancing policy and the consumption function.

Keywords: precautionary savings, employment risk, mortgages, housing

JEL No: D11, D14, E21

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\*Comments on an earlier version of this paper from participants at the Copenhagen Workshop on Household Choice of Consumption, Housing and Portfolios and seminars in Padua are gratefully acknowledged. We wish to thank Martin Browning, Chris Carroll and Guglielmo Weber for useful suggestions. The usual disclaimer applies.

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# 1 Introduction

Housing is one of the major spending and financial decisions facing consumers. In the UK the owner occupation rate is 70% (table 1) and housing is one of the few spending decisions that has a specific asset to assist its financing: the mortgage. In addition, for the majority of consumers wealth largely consists of housing and pension wealth. Since the latter is usually untouchable, the flexible part of wealth is primarily housing equity. Thus, our key argument is that most cash-convertible wealth increases are in housing wealth - pension wealth is not realisable and most households have quite negligible average values of financial assets. In the context of a life cycle model the realisation of wealth accruals is driven by the desire to smooth consumption. However, since consumers need somewhere to live, increases in housing wealth can only be realised to a limited extent, mainly in three ways: either by trading down house quality, by moving into rented accommodation or by remortgaging. In this paper we focus on the last one. Our view is reinforced by the policy concern over the importance of housing equity withdrawal as a means of financing consumption sprees, which may then lead to lack of macroeconomic control of the economy. Of course the reverse side of the coin is the prospect of falls in house prices and, thus, in housing wealth, in which case housing debt can lead to consumers being locked into negative equity. Therefore, housing wealth and equity are major drivers of consumption.

To analyse the effects of house price uncertainty and mortgage equity withdrawal on consumption, we use a real model with a time additive utility over a finite horizon and a positive rate of time preference. The individual has a portfolio composed of two financial assets: a mortgage and a riskless asset in which she can borrow and save. The mortgage is an adjustable rate

<b>Household tenure</b>	<b>%</b>
Rented	30%
Owner occupied	70%
<i>with mortgage</i>	<i>40%</i>
<i>owned outright</i>	<i>30%</i>

Table 1: Household tenure in the UK. Source: Office for National Statistics, 2003-2004

loan that is taken for the remaining life of the consumer but can be increased at any date. In the mortgage market there are two imperfections: refinancing the mortgage involves the payment of a fixed transaction cost and the new loan can never exceed the latest realised house price. In the last period of the life the house is sold and any outstanding debt must be paid off. In the model we include both employment and real house price risk, while wages and real interest rates are time varying and driven by stochastic processes, but they are foreseen by the individual

The new features of our framework (which in some respects is very stylised) are:

- We explicitly model housing equity withdrawal through remortgaging as house prices change in an environment in which decisions are taken under house price uncertainty and employment risk.
- There is no approximation in our approach; some variables are perfectly foreseen by the decision maker but others, such as house prices and employment status, are not, and despite this we derive a closed form analytical solution for optimal equity withdrawal and consumption.
- The result is a consumption function in which the current and random values of future house prices play a critical role. On the one hand,

uncertain future house prices give the opportunity of high prices and so the chance of high future equity withdrawal, reducing the need for current buffer stock savings against future employment uncertainty. On the other hand, there is a risk that future house prices might be low, constraining the refinancing possibilities, which serves to depress current consumption. Which of these two effects dominate depends on the interest rate differential and house price expectations.

- We also find a financial portfolio decision important in financing consumption. If we have a recontractible one period mortgage and a one period bond, then with no saving or borrowing constraints in the bond, there is an issue of whether the consumer should finance consumption from bond borrowing or from remortgaging when housing wealth increases. We show that, despite any degree of risk aversion, with the preferences that we use, consumers will choose the cheapest financial source to finance consumption smoothing as housing wealth rises.
- There are special end of life effects. Consumers cannot die in debt so any outstanding mortgage must be paid off by current realisable wealth. But there is a cleft stick: in the final period of life, house prices might jump to high levels leading to large unanticipated wealth increases, or they could fall leading to the need to find some financial way of paying off any existing mortgage debt outside of the value of the house.

The plan of the paper is to review the literature in section 2, give the assumptions in section 3 and derive the overall value function explicitly in sections 4 and 5. In section 6 we analytically derive the consumption function. We then briefly discuss extensions and conclude.

## 2 Literature Review

Housing impacts on individuals in various ways. One strand of the literature focuses on the role of housing on portfolio decisions. Some economies have a liquid rental market for housing and an illiquid retail market; others have a negligible rental market outside metropolitan areas but an active retail market (Chiuri and Jappelli, 2001). Flavin and Yamashita (2003) stress that with a thin rental market housing decisions have to balance financial asset portfolio considerations with the need for housing services. In the UK about 43% of homeowners bought their house outright without a mortgage (table 1) and thus they face the risk of random house prices - both nominal and real. Several authors have argued that this risk explains why consumers do not invest in equities but in relatively safe assets: their overall portfolio including housing has the right balance of risk and return without investing in equities (Pelizzon and Weber, 2003). On the other hand, 57% of house buyers do have a mortgage and for them there are additional risks: the liquidity risk that with a bad income shock it would be difficult to maintain repayments (Fratantoni, 2001) and the risk that the mortgage interest rate might change. In addition, they have to make other decisions: when to repay the mortgage or, in common with the first group who have no mortgage, when to withdraw equity from the house. Furthermore, both of these groups face common shocks, such as wage and employment risk. The importance of these different risks varies over the life cycle; usually for highly geared young households, with housing debt a high proportion of wealth and income (Cocco, 2005), the liquidity risk is higher than for older households with, on average, more diversified wealth and more house equity. This implies that the typical life cycle portfolio composition sees systematic changes in the share of housing in wealth.

There is also a recent literature that looks at the collateral/buffer stock effect of housing investment on precautionary savings. The argument is that, although the retail housing market is illiquid, housing equity serves as a buffer stock of wealth against low probability but very negative income shocks. Thus, even if housing is rarely traded because of the high transaction costs, it allows a higher level of mean consumption since if the worst income events occur there is a buffer stock of wealth that can be realised. This ignores the fact that most homeowners hold mortgage debt against their housing (Campbell and Cocco, 2003), so that net housing wealth may be quite low, and also the fact that consumers need a roof over their head, so that they might not be able to sell the house where they live and, even if they are, they have to buy or rent another one. In the presence of high transaction costs in housing and a thin rental market, consumers can withdraw equity from their properties by using the buffer stock of housing wealth as collateral against loans. This is predominantly executed by remortgaging. However, mortgage markets are imperfect in that there are costs in altering the debt position and limits on the amount that can be borrowed in a mortgage. This means that there may be inertia in the refinancing decision. For example, in the US, where most mortgages are at fixed nominal interest rates, there is active refinancing as interest rates change (Majumdar, 2004), but in Europe this is less common (Smith and Vass, 2004).

Another issue in the literature is related to the degree of substitutability of housing and financial wealth in determining consumption. Will two households with the same aggregate wealth, the same labour income prospects and the same preferences follow the same consumption function if one of them has a much higher proportion of housing wealth than the other? The empirical results here are mixed (Hoynes and McFadden, 1994; Bostic *et al.* 2004;

Majumdar, 2004). The question is important since consumer spending fluctuations are often seen as an important determinant of business cycle fluctuations. The main transmission mechanism for converting changes in housing wealth into disposable resources is the mortgage; therefore, analysing how mortgage decisions are made is crucial. The paper of Campbell and Cocco (2003) is a related major study of the relative advantages of fixed and variable rate mortgages that is close to our concerns. Li and Yao (2005) also study the effects of house price changes on household consumption. However, their analysis substantially differs from ours in that all mortgage refinances in their model are for consumption purposes only and are not driven by financial efficiency considerations; this arises from the assumption that the unsecured and secured borrowing rate are set to be equal.

With its complex features, modelling these housing choices is not straightforward. Quite special assumptions are necessary to derive analytical conclusions; for example Ortalo-Magné and Rady (2006) use utility linear in consumption, a fixed utility premium for a large house over a smaller one and no uncertainty and derive analytical results about the nature of general equilibrium house prices. Campbell and Cocco (2003), Cocco (2005), Li and Yao (2005) and Yang (2005) rely on calibrated numerical simulation involving grid search over both their state and control variables to derive the optimum. This approach allows the use of more general assumptions about the stochastic processes that drive house prices, interest rates and labour income but does not yield analytical conclusions.

### 3 The Model Assumptions

We consider a finite horizon  $T$  of discrete time periods  $t$ . Within a period  $t < T$  the timing of the model is as follows. At the start of the period the individual has a portfolio composed of two financial assets: a riskless asset  $A_t$  with rate of return  $r_t$  and a mortgage  $M_t$  that allows the investor to borrow against the value of the house at a rate  $\rho_t$ . We assume that both  $r_t$  and  $\rho_t$  are perfectly foreseen and that a mortgage loan initiated at  $t$  matures at  $T$ . In each period, consumers decide whether to refinance or not. When refinancing occurs, they redeem the existing debt, pay a transaction cost  $k$  and choose a new mortgage size  $M_{t+1}$ , which cannot exceed the latest realised price  $p_t$ . House prices are stationary with a deterministic trend. Then, consumption  $c_t$  is chosen and next the individual receives labour income  $w_t^s$ : if the individual has a job, this is the wage  $w_t$ , if she is unemployed, it is benefits  $B_t$ . Each period there is a constant probability  $a$  of being unemployed and wages and benefits are time dependent but perfectly foreseen. Finally, the stock of financial assets  $A_{t+1}$  to carry forward into the next period is determined. This means that assets bear all the effects of shocks in employment, but this only affects the current period and is insignificant over the lifetime<sup>1</sup>. Note also that as viewed from earlier periods  $M_{t+1}$  is random, since it depends on the realisation of random house prices through the constraint  $M_{t+1} \leq p_t$ .

The general form of the budget constraint in any period before the final one is

$$A_{t+1} = (1 + r_t)A_t + w_t^s + (1 - \rho_t)M_{t+1} - M_t - k - c_t$$

If there is no refinancing  $M_{t+1} = M_t$  and no transaction cost has to be paid

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<sup>1</sup>This is unimportant and arises from the definition of the period, but the use of this timing simplifies the analysis. You may think of shopping on Saturday and being paid the subsequent Friday.

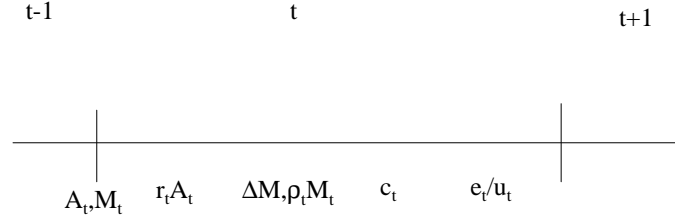


Figure 1: Timing

( $k = 0$ ):

$$A_{t+1} = (1 + r_t)A_t + w_t^s - \rho_t M_t - c_t$$

The consumer enters the final period with mortgage debt  $M_T$ , financial assets  $A_T$  and with a realised house price of  $p_T$ . At the period start the individual sells the house (but arranges to continue living in it for the duration of the period<sup>2</sup>), redeems any outstanding mortgage and consumes all the known cash on hand.

$$c_T = (1 + r_t)A_T + p_T + w_T^s - M_T - k$$

We also assume that in the last period for sure the individual is unemployed (but we could also think of this as retirement) so that  $w_T^s = B_T$ . This is without loss of generality since consumption is determined prior to knowledge of employment status and then in the last period it would have to be reined back to a level that will prove feasible if it turns out that the individual is unemployed, as it is impossible to die in debt.

Lifetime preferences are additive and there is a positive intertemporal

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<sup>2</sup>There is a small market in which equity in the house can be realised in the last years of life eg by selling the house to a financial institution and buying back an option to live in it until death but this is not very well developed.

discount rate  $\phi$ :<sup>3</sup>

$$U_0 = \sum \phi^t u(c_t)$$

The per-period utility function has a CARA form and depends only on consumption:

$$u(c_t) = 1 - \exp(-bc_t)$$

where  $b$  is the coefficient of absolute risk aversion. That is, there is a zero utility of housing and an inelastic labour supply. Since we assume that the individual keeps the house throughout her life, then omitting it from the value function is without loss of generality. Ignoring the disutility of work is more serious and is based on simplicity; we could include it assuming that jobs have fixed hours of work. Similarly, we could incorporate socio-demographic variables, such as the number of children into the analysis. CARA preferences have the advantage of exhibiting prudence and allowing us to derive exact solutions without having to approximate Euler equations. Much of the literature works with isoelastic felicity, which generally requires approximation to get solutions. On the one hand, there is some evidence (Gourinchas and Parker, 2002) that the error involved can be substantial; on the other hand, since isoelastic preferences have unbounded marginal utility at zero consumption, they generally serve to keep cash on hand positive and, therefore, almost act like a liquidity constraint. With CARA, marginal utility is finite at zero consumption, so we may expect to see the consumer actively borrowing. However, the lifetime budget constraint prevents her dying in debt.

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<sup>3</sup>It would be very simple to add a bequest motive, especially if the utility of bequests also has an exponential form.

The overall optimisation problem has the form

$$\begin{aligned} & \max \sum_{t=1}^T \phi^t [1 - \exp(-bc_t)] \\ & \text{subject to} \\ A_{t+1} &= (1 + r_t)A_t + w_t^s + (1 - \rho_t)M_{t+1} - M_t - k - c_t & M_{t+1} > M_t, \quad t < T \\ A_{t+1} &= (1 + r_t)A_t + w_t^s - \rho_t M_t - k - c_t & M_{t+1} = M_t, \quad t < T \\ c_T &= (1 + r_T)A_T + p_T + B_T - M_T - k \\ M_t &\leq M_{t+1} \leq p_t \end{aligned}$$

The interesting empirical questions about mortgage refinance concern equity withdrawal and portfolio diversification among housing assets, mortgage debt and net financial assets. There are two reasons primarily for mortgage refinancing. The first motivation is that bank borrowing might get very cheap relative to mortgage debt in which case the consumer may wish to switch from secured to unsecured debt so far as this is allowed. The second reason for refinancing is that as house prices and, thus, wealth rise, the individual may want to withdraw equity from the house by increasing the mortgage. To focus on the latter we impose the constraint that  $M_t \leq M_{t+1}$ . Our argument is that consumers will only wish to reduce their mortgage debt if there is an unanticipated income gain large enough so that consumption smoothing benefits overcome the transaction cost that has to be paid to reduce the mortgage. In practice in the reality this does not happen - it is confined to a tiny proportion of the population who receive large lump sum income gains, e.g. winning the national lottery or large bequests from wealthy relatives. Hence, we argue that the constraint  $M_t \leq M_{t+1} \leq p_t$  captures the essence of housing equity withdrawal combined with lender limits on the extent to which this is possible.

## 4 Value Function

In any period  $t < T$  the value function is the maximum of the value function with refinancing and the value function without any mortgage refinancing:

$$V_t = \max \{V_t^R, V_t^{NR}\}$$

Based on Merton (1992), and Berloffia and Simmons (2003), we conjecture that  $V_t$  takes the form:

$$V_t(A_t, M_t, p_t) = \alpha_t - \beta_t \exp \{-b\delta_t [(1 + r_t) A_t - M_t]\} \quad (1)$$

The first main result of our paper is to derive this form and the functions  $\alpha, \beta, \delta$ . In fact,  $\alpha$  will prove to be a discounting function depending on the rate of time preference and  $\delta$  a discounting function depending on interest rates. The most interesting function is  $\beta$ , which depends on the expectation of future house prices and employment states and on the future optimal mortgage and saving decisions.

In periods before the final one (in which refinancing does not occur) the form of the value function depends on whether it is optimal to undertake refinancing. We derive the value functions at  $t$  with and without refinancing and then compare them to determine the optimal refinancing decision.

The appendix shows that conditional on the refinancing decision, the value function with refinancing is

$$\begin{aligned} V_t^R(A_t) = & 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1 + r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ & \cdot \exp \{-b\Delta_t[(1 + r_t) A_t - M_t]\} W_t \\ & \cdot \exp \{-b\Delta_t[((1 - \rho_t) M_{t+1} - k) - M_{t+1}/(1 + r_{t+1})]\} / \Delta_t \end{aligned}$$

where

$$W_t = \{a \exp[-b\delta_{t+1}(1+r_{t+1})w_t] + (1-a) \exp[-b\delta_{t+1}(1+r_{t+1})B_t]\}^{1/[1+\delta_{t+1}(1+r_{t+1})]}$$

is the expected utility term corresponding to next periods labour income and:

$$\Delta_t = \frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}$$

is the "discounted future interest rate".

At the start of  $t$ , given that remortgaging takes place, the mortgage refinancing decision is to choose  $M_{t+1}$  to maximise  $V_t^R(A_t)$  within the constraint  $M_t \leq M_{t+1} \leq p_t$ . Defining  $\lambda_t = (1-\rho_t) - 1/(1+r_{t+1})$ , this is equivalent to minimising:

$$\exp(-b\Delta_t M_{t+1} \lambda_t)$$

The decision rule is then:

$$\begin{aligned} M_{t+1} &= p_t & \text{if } \lambda_t > 0 \\ M_{t+1} &= M_t & \text{if } \lambda_t < 0 \end{aligned}$$

Therefore, refinancing occurs only when  $\lambda_t > 0$  and the individual chooses maximum equity withdrawal. Integrating this into the value function

$$\begin{aligned} V_t^R(A_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \cdot \\ &\quad \cdot \exp\{-b\Delta_t[(1+r_t)A_t - M_t]\} \cdot \exp\left\{-b\Delta_t \left[\max(\lambda_t, 0) \left(p_t - \frac{k}{\lambda_t}\right)\right]\right\} \\ &\quad \cdot \exp\left\{b\Delta_t \left[\max(-\lambda_t, 0) \left(M_t - \frac{k}{\lambda_t}\right)\right]\right\} \cdot W_t/\Delta_t \end{aligned}$$

Similar arguments show that without refinancing

$$V_t^{NR}(A_t) = 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ \cdot \exp\{-b\Delta_t[(1+r_t)A_t - M_t]\} \cdot \exp(-b\Delta_t\lambda_t M_t) \cdot W_t/\Delta_t$$

By comparing the two value functions we obtain the condition under which refinancing occurs. At  $t$  the individual chooses to refinance the mortgage if  $V_t^R(A_t) > V_t^{NR}(A_t)$ .

If  $\lambda_t > 0$  then refinancing occurs when

$$p_t - M_t > \frac{k}{\lambda_t}$$

In terms of the debt service costs it pays to refinance to the highest extent possible by setting  $M_{t+1} = p_t$  so long as the interest gain on the sum involved more than covers the transaction cost of refinancing.

On the other hand, if  $\lambda_t < 0$  (the mortgage interest rate at  $t$  is relatively high with respect to  $r_{t+1}$ ), no refinancing occurs.

We can summarise the possible remortgage actions at time  $t$  as

$\lambda_t > 0$	$M_t = p_{t-s}$	$\lambda_t p_t > \lambda_t p_{t-s} + k$	$M_{t+1} = p_t$
		$\lambda_t p_t < \lambda_t p_{t-s} + k$	$M_{t+1} = p_{t-s}$
$\lambda_t < 0$	$M_t = p_{t-s}$	$\lambda_t p_{t-s} + k > 0$	$M_{t+1} = p_{t-s}$

Table 2: Possible remortgage actions

## 5 Overall Value Function

The overall value function is the larger of  $V_t^R(A_t, M_t)$  and  $V_t^{NR}(A_t, M_t)$ , which gives us the result

**Proposition 1** *The value function has the form<sup>4</sup>*

$$V_t(A_t, M_t, p_t) = \alpha_t - \beta_t \exp \{-b\delta_t [(1 + r_t) A_t - M_t]\}$$

where  $\alpha_t, \delta_t, \beta_t$  are defined recursively through

$$\alpha_t = 1 + \phi\alpha_{t+1} \quad (2)$$

$$\delta_t = \Delta_t = \frac{\delta_{t+1}(1 + r_{t+1})}{1 + \delta_{t+1}(1 + r_{t+1})} \quad (3)$$

$$\beta_t = \left[ \phi(E\beta_{t+1}) \frac{\delta_t}{1 - \delta_t} \right]^{1 - \delta_t} W_t / \delta_t \exp [b\Delta_t \max(-\lambda_t, 0) M_t] \quad (4)$$

$$\cdot \exp \left\{ -b\delta_t \left[ \max(\lambda_t, 0) \cdot \max \left( p_t - \frac{k}{\lambda_t}, M_t \right) \right] \right\}$$

Solving the recurrence relations for  $\alpha_t$  and  $\delta_t$

$$a_{T-t} = \sum_{s=0}^t \phi^s$$

$$\delta_{T-t} = \frac{\prod_{s=0}^{t-1} (1 + r_{T-s})}{1 + \sum_{\tau=0}^{t-1} \prod_{s=0}^{\tau} (1 + r_{T-s})}$$

The recurrence relation in  $\beta$ , which captures the effect of future house prices and employment uncertainty, requires some careful analysis.

## 5.1 $E\beta_{t+1}$ : The Effects of Future House Price Uncertainty

The effects of future house price uncertainty at time  $t$  work through the expression  $E\beta_{t+1}$ . Since we have assumed that any trend in  $p_t$  is not stochastic,

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<sup>4</sup>In fact the value function has this same form (with a different definition of  $\beta$ ) if  $\rho$  is stochastic and  $k$  is time varying or stochastic as long as the current period mortgage rate and transaction cost are realised prior to the current refinancing decision.

$E_{t+1}\beta_{t+2}$  is not a function of  $p_{t+1}$  and at time  $t$  the random term in  $\beta_{t+1}$  is

$$\begin{aligned} F_{t+1} &= \exp \left\{ -b\delta_{t+1} \left[ \max(\lambda_{t+1}, 0) \cdot \max \left( p_{t+1} - \frac{k}{\lambda_{t+1}}, M_{t+1} \right) \right] \right\} \quad (5) \\ &= \exp(-b\delta_{t+1}G_{t+1}) \end{aligned}$$

where

$$\begin{aligned} G_{t+1} &= 0 \text{ if } \lambda_{t+1} < 0 \\ &= \lambda_{t+1}M_{t+1} \text{ if } \lambda_{t+1} > 0 \text{ and } p_{t+1}\lambda_{t+1} - k < \lambda_{t+1}M_{t+1} \quad (6) \end{aligned}$$

$$= p_{t+1}\lambda_{t+1} - k \text{ if } \lambda_{t+1} > 0 \text{ and } p_{t+1}\lambda_{t+1} - k > \lambda_{t+1}M_{t+1} \quad (7)$$

We want to compute  $EF_{t+1}$  over  $p_{t+1}$ .

If  $\lambda_{t+1} < 0$ , the consumer does not wish to refinance. In this case  $G_{t+1}$  and the conditions are not random, so  $EF_{t+1} = F_{t+1}$ .

If  $\lambda_{t+1} > 0$  it is more complex. The second case (6) holds when house prices are such that the consumer potentially wishes to refinance to the maximum permissible extent but the savings from doing so will not cover the transaction cost of refinance. This occurs when

$$p_{t+1} < M_{t+1} + \frac{k}{\lambda_{t+1}}$$

On the contrary, in the third case (7), which holds for  $p_{t+1}$  above this critical value, the gains from taking out a new maximum mortgage do cover the transaction cost. Hence when  $\lambda_{t+1} > 0$ , if we define the probability that the maximum remortgage will not cover the transaction cost by  $\gamma_t = \Pr(p_{t+1} <$

$M_{t+1} + \frac{k}{\lambda_{t+1}}$ ), the expression for  $EF_{t+1}$  becomes:

$$EF_{t+1} = E \left\{ \exp[-b\delta_{t+1}(p_{t+1}\lambda_{t+1} - k)] \left| p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}} \right. \right\} (1 - \gamma_t) \\ + \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})\gamma_t$$

Suppose that the support of the distribution of  $p_{t+1}$  is  $[\underline{p}_{t+1}, \bar{p}_{t+1}]$ :

$$\text{if } M_{t+1} + \frac{k}{\lambda_{t+1}} < \underline{p}_{t+1} \quad \text{then } \gamma_t = 0 \\ \text{if } M_{t+1} + \frac{k}{\lambda_{t+1}} > \bar{p}_{t+1} \quad \text{then } \gamma_t = 1$$

Therefore, we have two boundary cases where either house prices are always so low that a maximum equity withdrawal will not cover the transaction cost or they are so high that with certainty the maximum remortgage is profitable. Table 2 summarises the various forms of  $EF_{t+1}$ . Since

$$E\beta_{t+1} = \left[ \phi(E\beta_{t+2}) \frac{\delta_{t+1}}{1 - \delta_{t+1}} \right]^{1 - \delta_{t+1}} \cdot EF_{t+1} W_{t+1} / \delta_{t+1} \quad (8)$$

the nature of  $E\beta_{t+1}$  follows. Thus, there are only effects of immediate future house price uncertainty on the current value function if  $\lambda_{t+1} > 0$ . In fact, extending this argument, the distribution of house prices at any future date  $\tau$  only affects the current value function for those periods in which  $\lambda_\tau > 0$ .

Appendix A.2 shows that<sup>5</sup>:

$$\begin{aligned}
\beta_t = & (E_{T-1}\beta_T)^{(1-\delta_t)(1-\delta_{t+1})\dots(1-\delta_{T-1})} & (9) \\
& \left(\frac{\phi}{1-\delta_t}\right)^{1-\delta_t} \left(\frac{\phi}{1-\delta_{t+1}}\right)^{(1-\delta_t)(1-\delta_{t+1})} \dots \left(\frac{\phi}{1-\delta_{T-1}}\right)^{(1-\delta_t)(1-\delta_{t+1})\dots(1-\delta_{T-1})} \\
& F_t (E_t F_{t+1})^{1-\delta_t} (E_{t+1} F_{t+2})^{(1-\delta_t)(1-\delta_{t+1})} \dots (E_{T-2} F_{T-1})^{(1-\delta_t)(1-\delta_{t+1})\dots(1-\delta_{T-1})} \\
& W_t (W_{t+1})^{1-\delta_t} \dots (W_{T-1})^{(1-\delta_t)(1-\delta_{t+1})\dots(1-\delta_{T-1})} \\
& \delta_t^{-\delta_t} \delta_{t+1}^{-\delta_{t+1}(1-\delta_t)} \dots \delta_{T-1}^{-\delta_{T-1}(1-\delta_t)(1-\delta_{t+1})\dots(1-\delta_{T-1})}
\end{aligned}$$

Note that there is an effect on  $\beta_t$  of the time to go to the horizon: the longer the remaining future, the higher the number of terms in the product for  $\beta$  since there are more future nodes. Therefore, in earlier periods  $\beta$  tends to be higher, which reflects the effect of the greater amount of uncertainty remaining. Conversely, towards the end of life there is little remaining uncertainty and so on these grounds less of a need for precautionary savings.

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<sup>5</sup> If the probability of unemployment was either history dependent or uncertain, then so long as it is independent of house prices there is little impact on the expression for  $\beta$ : terms in  $W$  would become  $E_t W_{t+1}$ .

	$\mathbf{EF}_{t+1}$
$\lambda_{t+1} < 0$	1
$\lambda_{t+1} > 0$	$E \exp [-b\delta_{t+1} (p_{t+1}\lambda_{t+1} - k)]$
	$\exp (-b\delta_{t+1}\lambda_{t+1}M_{t+1})$
$p_{\underline{t+1}} < M_{t+1} + \frac{k}{\lambda_{t+1}} < \bar{p}_{t+1}$	$\exp (-b\delta_{t+1}\lambda_{t+1}M_{t+1}) \gamma_t + E \left\{ \exp [-b\delta_{t+1} (p_{t+1}\lambda_{t+1} - k)] \left[ p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}} \right] (1 - \gamma_t) \right\}$

Table 3:  $EF_{t+1}$

## 6 Consumption

From the relevant first order conditions for consumption we can derive the consumption function and examine its properties.

**Proposition 2** *Optimal consumption is given by*

$$c_t = -\frac{1-\delta_t}{b} \ln \left[ \frac{\phi(E\beta_{t+1})}{1-\delta_t} \right] + \delta_t [(1+r_t)A_t - M_t] - \frac{1-\delta_t}{b} \ln(W_t) \\ + \delta_t \left[ \max(\lambda_t, 0) \cdot \max\left(p_t - \frac{k}{\lambda_t}, M_t\right) - \max(-\lambda_t, 0) M_t \right]$$

This expression subsumes two cases: when  $\lambda_t > 0$  and  $p_t\lambda_t - k > M_t\lambda_t$  the individual chooses to refinance, whereas when  $\lambda_t > 0$  and  $p_t\lambda_t - k < M_t\lambda_t$  or  $\lambda_t < 0$  optimally there is no refinance. The main features of the consumption function are summarised by proposition 3.

**Proposition 3** *Uncertainty of future house prices may raise or lower current consumption. If*

(a)  $\lambda_{t+1} > 0$  and  $\underline{p}_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$  *optimal consumption under uncertain house prices is lower than with house price certainty.*

(b)  $\lambda_{t+1} > 0$  and  $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$  *uncertainty in house prices may raise or lower consumption depending on the nature of the house price distribution, the interest rates and the degree of risk aversion*

(c)  $\lambda_{t+1} > 0$  and  $Ep_{t+1}\lambda_{t+1} - k < M_{t+1}\lambda_{t+1}$  *optimal consumption under uncertain house prices is higher than with house price certainty and precautionary savings are negative.*

**Proof.** See the appendix. ■

Both with and without refinancing the effect of future period employment risk and house price uncertainty is to shift the intercept of the consumption

function by an amount that depends on the degree of risk aversion. Since it is possible that  $\phi(E\beta_{t+1}) < 1 - \delta_t$ , the future uncertainty in house prices may actually increase rather than reduce consumption. Uncertain future house prices give the opportunity of high prices and so the chance of high future equity withdrawal, reducing the need for current buffer stock savings against future employment uncertainty. If future house prices are certain, then the expression for  $E\beta_{t+1}$  changes when  $\lambda_{t+1} > 0$ . We can gauge the effect of this by comparing  $E\beta_{t+1}$  with its value when house prices are constant at their mean  $Ep_{t+1}$ . In (a) with both certain and uncertain house prices the consumer has the same expected refinancing possibilities; this is the standard case in which uncertainty lowers current consumption unambiguously and precautionary savings are positive. In (b) on the one hand there is a risk that future house prices might be low, constraining the refinancing possibilities, which serves to depress current consumption; on the other hand, house prices might be high, giving the opportunity of future equity withdrawal, higher than with certain house prices. The overall effect is ambiguous and depends on future house price expectations. The ambiguity exists even though we do not have any stochastic trend in house prices. If we had, this result would be even stronger. Finally in (c), when with certain future house prices no refinancing is undertaken ( $Ep_{t+1}\lambda_{t+1} - k < M_{t+1}\lambda_{t+1}$ ), precautionary savings are negative and house price uncertainty serves to raise current consumption. The intuition is that in this case house price risk is just upside risk.

Summing up, in each of these cases (a)-(c) housing is acting like an intertemporal buffer stock of wealth with effects on the current level of savings. Knowing that in the future there will be a redeemable asset (though of uncertain value), the consumer can afford to borrow today. In addition, the composition of wealth between net housing wealth ( $p_t - M_t$ ) and  $A_t$  has an

impact on consumption. The marginal propensity to consume out of wealth is  $\delta_t(1 + r_t)$  but out of a reduction of the current mortgage is  $\delta_t(1 - \lambda_t)$ .

Moreover, there is an effect on consumption of the remaining length of the horizon. We have seen that  $E\beta_t$  tends to fall through time and this serves to yield consumption growth through time *ceteris paribus*. As time passes there is less remaining uncertainty and thus less of a need for precautionary savings.

**Proposition 4** *Labour income uncertainty depresses consumption and raises savings.*

**Proof.** See the appendix. ■

As usual with CARA preferences, consumption is linear in net wealth  $(1 + r_t)A_t - M_t$  but is nonlinear in expected current period labour income. The latter is essentially an artifact of the assumed within period timing where consumption has to be chosen before the employment status of the current period is realised. In fact this labour income risk reduces consumption for familiar reasons due to the risk aversion of the consumer. For current labour income this works through the expected utility of current labour earnings, for future labour income risk it works through the concavity of  $E\beta_{t+1}$ .

**Remark 5** *When it is optimal to refinance, there is maximum equity withdrawal ( $M_{t+1} = p_t$ ) and consumption increases*

When it is optimal to refinance, the effect of equity withdrawal on current consumption is unambiguously non-negative. However, with no refinancing the current mortgage state may increase or reduce consumption. It will have a negative effect on consumption when  $\lambda_t < 0$  and a positive one when  $\lambda_t > 0$  but the cost saving from increasing the current mortgage  $M_t$  to  $p_t$  does not cover the transaction cost of doing so.

**Remark 6** *Current mortgage rates influence consumption through  $\lambda_t$ : a fall in the mortgage rate encourages refinancing which raises consumption.*

The current mortgage interest rate only has effects via  $\lambda_t$ . When  $\lambda_t > 0$  and  $p_t > M_t$  there is scope for increasing the mortgage so long as the interest gain covers the transaction cost. A fall in the current mortgage rate increases the chance of refinancing and therefore may cause a switch from no refinancing of the current mortgage to refinancing to the maximum extent possible, which causes a jump in consumption. Only a part of the current wealth change arising from remortgaging is consumed in the current period (the coefficient  $\delta$ ), while the other part of the wealth change is used to smooth future consumption.

**Remark 7** *The current savings interest rate has income effects on consumption, the effects of future interest rates on savings are complex.*

The current saving interest rate has obvious income effects on consumption. First, there is a consumption increasing effect through raising capital income when the consumer has positive financial assets, but a decreasing effect through raising the debt service cost when assets are negative. Future savings rates and especially that of the next period have much more complex effects: directly through altering the slope of the consumption function in most variables, indirectly through varying  $\lambda_t$  and affecting the discounting terms in  $E\beta_{t+1}$ . If at  $t$  the foreseen  $r_{t+s}$  increases ( $s > 0$ ) all the terms in  $\delta_\tau$  for  $t \leq \tau < s$  increase. In particular if  $s = 1$  then  $\delta_{t+1}$  is unaffected and  $1/(1 + \delta_{t+1}(1 + r_{t+1}))$  falls, so that the marginal propensity to consume out of labour income expected for this period decreases.

**Remark 8** *An increase in risk aversion reduces the variability of consumption over time.*

As the degree of risk aversion rises, the absolute value of the intercept of the consumption function falls in any period and so between any two periods there is less variability in consumption. In particular there are smaller jumps in consumption when refinancing occurs.

## 7 Extensions

Our approach suggests some obvious areas for future research and has various special assumptions whose force we try to evaluate in this section.

First, we have assumed no liquidity constraints in the financial asset: as long as the lifetime budget constraint is respected, consumers can borrow as much as they wish. This assumption is important in making the key determinant of the refinancing decision the relative interest rate advantages of borrowing against the house or against future wealth (including future labour earnings and the future value of the house). An alternative is to impose the constraint  $A_t \geq 0$  but the refinancing decision becomes much less transparent since it has to take account of the fact that remortgaging now influences the chance with which in the next period the consumer may end up being liquidity constrained, e.g. if she loses her job.

Second, generally the amount that can be borrowed on a mortgage is limited not only by the house value but also by the current income level. The rationale for this constraint seems to be on debt service cost grounds. For example in the UK the mortgage cannot usually exceed three or four times the income. However, to incorporate the income constraint into the analytical framework would raise nothing new conceptually and it would make the algebra more complicated. Given that the individual wishes to

refinance in period  $t$ , the new mortgage decision would be<sup>6</sup>

$$M_{t+1} = \min(p_t, \mu w_{t-1}) \quad \text{if} \quad \lambda_t > 0$$

Again it is optimal to remortgage only if there is a financial advantage:

$$\min(p_t, \mu w_{t-1}) \max(\lambda_t, 0) - k - M_t \lambda_t > 0$$

By substituting this condition into the value function, we obtain the term in mortgage activity:

$$\min \{ \exp(-b\Delta_t \lambda_t M_t), \exp \{ -b\Delta_t (1 + r_{t+1}) [\max(\lambda_t \min(p_t, \mu w_{t-1}), 0) - k] \} \} W_t / \Delta_t$$

Even in this case all the effects of uncertainty are captured in  $\beta$ , whose recurrence relation (4) becomes

$$\beta_t = \left[ \phi(E\beta_{t+1}) \frac{\delta_t}{1 - \delta_t} \right]^{1-\delta_t} \exp \{ -b\delta_t \max[\lambda_t M_t, \max(\lambda_t \min(p_t, \mu w_{t-1}), 0) - k] \} W_t / \delta_t$$

Then the time path of  $\beta$  can be easily deduced by following the methods of section 5.1.

Another obvious extension would be to allow for more than one type of house, e.g. a large expensive house with price  $p_t$  and a small cheaper house with price  $\pi_t$ , where both prices are uncertain and  $\pi_t < p_t$ . Consumers could then trade down from large to small houses and vice versa. In terms of housing decisions, at  $t$  there are three choices: retain the existing house and mortgage, retain the existing house but refinance, change house and refinance. In this context it makes sense to add a second transaction cost  $k_t^h$

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<sup>6</sup>Since the wage at  $t$  is unknown at the time of refinancing, the income constraint works on  $w_{t-1}$ .

that is incurred when changing house (in addition to the refinancing transaction cost). The effect is to add a third branch to the value function and the overall value function is then the maximum over the three branches. If we keep the other assumptions (especially that of no liquidity constraints), the refinancing decision will have the same form, once any house purchase/sale has been decided. Again all uncertainty will be channelled through  $\beta$ , the value function will have a similar structure and the recurrence relation for  $\beta$  (4) will become

$$\begin{aligned} \beta_t = & [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\ & \cdot \exp\{-b\delta_t \max[p_t - \pi_t - k_t^h + \pi_t \max(\lambda_t, 0) - k, \\ & \lambda_t M_t, p_t \max(\lambda_t, 0) - k]\} W_t / \Delta_t \end{aligned}$$

Furthermore, in the compulsory retirement case we could make the date of retirement uncertain. This is similar to dropping the perfect foresight assumption on wages and has significant effects (see Berloffia and Simmons, 2003, and below).

The main special assumptions that we have made are:

- CARA preferences depending only on consumption and independent of housing or leisure or socio-demographics.

A problem with CARA is that optimal consumption may turn out to be negative since marginal utility is finite at zero consumption. We could make the utility vary with housing and leisure; in the context of a model with a single indivisible house type, the former would add little, but the latter would be interesting and, although most of the structure of the value function, the refinancing decision and consumption would remain unchanged, there would be some additional preference effects

(see Berloffia and Simmons, 2003).

- Perfect foresight of unemployment benefit, wage and interest rates.

This is an important simplification with potentially large implications. If interest rates are uncertain, the consumer has a real portfolio choice, not just a choice driven by the asset with the highest return. We might then expect to get some diversification of the portfolio depending on the covariance between the interest rates and the covariance between interest rates and house prices.

Uncertainty in the real wage when employed can readily be incorporated so long as it is uncorrelated with house prices and with the chance of having a job. The value function, the refinancing decision and consumption will have a similar functional structure where the expected labour income term  $W_t$  becomes

$$W_t = aE_t \exp(-b\delta_{t+1} (1 + r_{t+1}) w_t) + (1 - a) \exp(-b\delta_{t+1} (1 + r_{t+1}) B_t)$$

If there is correlation between house prices and wages it is more complex. Another assumption we have made is that the real house price is stationary; we could relax this, allowing for a random walk with drift, and get similar results.

- A chance of being jobless that is constant (and foreseen) through time.  
It would be easy to allow the chance of unemployment to be time dependent - essentially it is just a notational change, the effect will be absorbed into  $W_t$ , so all the results will carry through.
- Omission of the impact of the tax system.

The treatment of interest income and payments, capital gains on housing and of implicit user services of owned housing differs between tax systems, so it is important to interpret these variables as post-tax.

- A single one period financial asset with a perfect capital market.

The biggest omission is the role of voluntary or involuntary contributions to pension schemes. Pension wealth can be defined by either the value of accumulated contributions to date or by the estimated pension income that will accrue at maturity. With the former approach and using the Family Resources Survey, Warren *et al.* (2001) find that individual median wealth was about £63k, which decomposed into median wealth of pensions £26k, financial wealth £1k and housing wealth £24k. The pension wealth divided into about 39% in state pensions, 53% in occupational pensions and only 8% in "discretionary" pensions. This asset structure accords with that found by others where liquid or risky financial assets are an insignificant proportion of individual wealth. Pension wealth is clearly important and serves both to remove some effects of uncertain date of death and to act as a buffer against asset shocks, e.g. falling real house prices late in life.

A further factor is that in reality there are wedges between the savings rate and the borrowing rate in bond type finance. Including this would affect the refinancing decision.

## 8 Conclusions

This paper uses a framework that allows us to analytically solve for the value function and the optimal lifecycle policies for consumption, saving in

financial assets and mortgage debt when a finitely lived, risk averse individual faces employment risk and uncertainty of house prices. This approach gives the advantage of being able to derive general propositions as opposed to specific simulation results without resorting to approximations which may have substantial inaccuracy. The financial asset market is perfect and up to 100% variable rate mortgages are allowed. Using CARA preferences and the assumption that house prices do not have stochastic trends while interest rates are certain and wages are foreseen facilitate explicit solution. However, the form of value function and optimal policy that we find is generally robust to relaxation of these special assumptions, similar results would follow if we had more constraints on available mortgages, uncertain wages, preferences depending not only on consumption but also on housing services, more than one type of house. One main result in all these cases is that there is a single "sufficient statistic" through which the effects of uncertainty on the value function and the optimal policies are channelled. This is due to the CARA form of preferences.

In terms of detail, we find that consumption is linear in wealth with an intercept that depends on future employment and house price risk, and a slope that depends on risk aversion and interest rates. The effects of housing wealth on consumption and saving/borrowing decisions primarily work through the mortgage. Consequently, the analysis of the refinancing of mortgages is important to understand how housing wealth can act as a buffer stock against bad shocks, e.g. in employment. We find that without liquidity constraints and with foreseeable interest rates, the refinancing decision is driven by financial efficiency considerations. The individual will refinance to the maximum extent possible in those periods in which the financial gains from doing so cover the transaction cost. Housing wealth and mortgage finance impact on

consumption so that in periods when it is optimal to refinance consumption jumps due to equity withdrawal. The financial gains from refinance are used partly to finance present and partly future consumption. Therefore, sometimes consumption tracks cash on hand and is not fully smoothed. In other periods there is an ambiguous effect of mortgage debt on consumption.

House price uncertainty may raise or reduce current consumption. On the one hand, the opportunity of high house prices gives the chance of high equity withdrawal. On the other hand, there is a risk that house prices might fall, constraining the refinancing possibilities, which serves to depress consumption. The overall effect is ambiguous and depends on the interest rate differential and house price expectations.

# A Appendix

## A.1 Value function

At  $T$  all available wealth is consumed and no mortgage interest is paid since no new mortgage debt is contracted so

$$0 = (1 + r_t) A_T + p_T - M_T - c_T + B_T - k$$

In the last period the value function is:

$$V_T = 1 - \exp \{-b [(1 + r_T) A_T - M_T]\} \exp(-bB_T) \exp[-b(p_T - k)]$$

This result is obtained simply substituting the budget constraint at  $T$  into the instantaneous CARA utility function. Hence, at  $T$

$$\begin{aligned} \alpha_T &= 1 \\ \beta_T &= \exp(-bB_T) \exp[-b(p_T - k)] \\ \delta_T &= 1 \end{aligned}$$

In any period before the final one, with refinancing the Bellman's equation says that  $c_t$  is determined to :

$$\max_{c_t} \{u(c_t) + \phi EV_{t+1}(A_{t+1}) | A_{t+1} = (1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - c_t + w_t^s - k\}$$

That is equivalent to:

$$\begin{aligned} \max_{c_t} \{u(c_t) + \phi E \{ \alpha_{t+1} - \beta_{t+1} \exp[-b\delta_{t+1} ((1 + r_{t+1}) A_{t+1} - M_{t+1})] \} \\ | A_{t+1} = (1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - c_t + w_t^s - k \} \end{aligned}$$

or

$$\max_{c_t} \{1 - \exp(-bc_t) + \phi\{\alpha_{t+1} - E\{\beta_{t+1} \exp[-b\delta_{t+1} (1 + r_{t+1}) ((1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - c_t + w_t^s - k) - M_{t+1}]\}\}\}$$

Since we are assuming that the risk of unemployment is foreseen and is independent of the uncertain house prices:

$$\begin{aligned} & E \{ \beta_{t+1} \exp [-b\delta_{t+1} (1 + r_{t+1}) w_t^s] \} \\ = & E \beta_{t+1} \{ a \exp [-b\delta_{t+1} (1 + r_{t+1}) w_t] + (1 - a) \exp [-b\delta_{t+1} (1 + r_{t+1}) B_t] \} \end{aligned}$$

The first order condition gives:

$$\begin{aligned} \exp(-bc_t) &= [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})] \exp [b\delta_{t+1} (1 + r_{t+1}) c_t] \exp (b\delta_{t+1} M_{t+1}) \\ &\cdot \exp \{-b\delta_{t+1} (1 + r_{t+1}) [(1 + r_t) A_t + (1 - \rho_t) M_{t+1} - M_t - k]\} \\ &\cdot \{a \exp [-b\delta_{t+1} (1 + r_{t+1}) w_t] + (1 - a) \exp [-b\delta_{t+1} (1 + r_{t+1}) B_t]\} \end{aligned}$$

that implies

$$\begin{aligned} \exp(-bc_t) &= [\phi(E\beta_{t+1})\delta_{t+1} (1 + r_{t+1})] \exp \{-b\delta_{t+1} (1 + r_{t+1}) [(1 + r_t) A_t - M_t]\} \\ &\cdot \exp [b\delta_{t+1} (1 + r_{t+1}) c_t] \exp (b\delta_{t+1} M_{t+1}) \\ &\cdot \exp \{-b\delta_{t+1} (1 + r_{t+1}) [(1 - \rho_t) M_{t+1} - k]\} \\ &\cdot \{a \exp [-b\delta_{t+1} (1 + r_{t+1}) w_t] + (1 - a) \exp [-b\delta_{t+1} (1 + r_{t+1}) B_t]\} \end{aligned}$$

Hence:

$$\begin{aligned}
\exp(-bc_t) &= [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} & (10) \\
&\cdot \exp\left\{b\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}[(1+r_t)A_t - M_t]\right\} \\
&\cdot \exp\left\{b\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}\left[(1-\rho_t)M_{t+1} - k - \frac{M_{t+1}}{1+r_{t+1}}\right]\right\} \\
&\cdot \{a \exp[-b\delta_{t+1}(1+r_{t+1})w_t] \\
&+ (1-a) \exp[-b\delta_{t+1}(1+r_{t+1})B_t]\}^{1/[1+\delta_{t+1}(1+r_{t+1})]}
\end{aligned}$$

and

$$\begin{aligned}
\exp[b\delta_{t+1}(1+r_{t+1})c_t] &= [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{-\delta_{t+1}(1+r_{t+1})/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\cdot \exp\left\{b\frac{[\delta_{t+1}(1+r_{t+1})]^2}{1+\delta_{t+1}(1+r_{t+1})}[(1+r_t)A_t - M_t]\right\} \\
&\cdot \exp\left\{b\frac{[\delta_{t+1}(1+r_{t+1})]^2}{1+\delta_{t+1}(1+r_{t+1})}\left[(1-\rho_t)M_{t+1} - k - \frac{M_{t+1}}{1+r_{t+1}}\right]\right\} \\
&\cdot \exp\left\{b\frac{\delta_{t+1}(1+r_{t+1})^2}{1+\delta_{t+1}(1+r_{t+1})}[(1-\rho_t)M_{t+1} - k]\right\} \\
&\cdot \{a \exp[-b\delta_{t+1}(1+r_{t+1})w_t] \\
&+ (1-a) \exp[-b\delta_{t+1}(1+r_{t+1})B_t]\}^{-\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}}
\end{aligned}$$

Therefore, conditional on the refinancing decision, taking expectations over the employment status at  $t$  the value function with refinancing is

$$\begin{aligned}
V_t^R(A_t) &= 1 + \phi\alpha_{t+1} - [\phi(E\beta_{t+1})\delta_{t+1}(1+r_{t+1})]^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\cdot \exp\left\{-b\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}[(1+r_t)A_t - M_t]\right\} \\
&\cdot \{a \exp[-b\delta_{t+1}(1+r_{t+1})w_t] \\
&+ (1-a) \exp[-b\delta_{t+1}(1+r_{t+1})B_t]\}^{1/[1+\delta_{t+1}(1+r_{t+1})]} \\
&\cdot \exp\left\{-b\frac{\delta_{t+1}(1+r_{t+1})}{1+\delta_{t+1}(1+r_{t+1})}\left[(1-\rho_t)M_{t+1} - k - \frac{M_{t+1}}{1+r_{t+1}}\right]\right\} \\
&\cdot \left[\frac{1+\delta_{t+1}(1+r_{t+1})}{\delta_{t+1}(1+r_{t+1})}\right]
\end{aligned}$$

## A.2 $E_t\beta_{t+1}$

Since there is no intertemporal stochastic dependence in house prices, from (8) it follows that:

$$\begin{aligned}
E_t\beta_{t+1} &= E_t\left\{(E_{t+1}\beta_{t+2})^{1-\delta_{t+1}}\left[\frac{\phi\delta_{t+1}}{1-\delta_{t+1}}\right]^{1-\delta_{t+1}}F_{t+1}W_{t+1}/\delta_{t+1}\right\} \\
&= (E_{t+1}\beta_{t+2})^{1-\delta_{t+1}}\left[\frac{\phi}{1-\delta_{t+1}}\right]^{1-\delta_{t+1}}\delta_{t+1}^{-\delta_{t+1}}(E_tF_{t+1})W_{t+1}
\end{aligned}$$

This equation can be solved recursively to get

$$\begin{aligned}
E_t\beta_{t+1} &= (E_{T-1}\beta_T)^{(1-\delta_{t+1})(1-\delta_{t+2})\dots(1-\delta_{T-1})} \\
&\left(\frac{\phi}{1-\delta_{t+1}}\right)^{1-\delta_{t+1}}\left(\frac{\phi}{1-\delta_{t+2}}\right)^{(1-\delta_{t+1})(1-\delta_{t+2})}\dots\left(\frac{\phi}{1-\delta_{T-1}}\right)^{(1-\delta_{t+1})\dots(1-\delta_{T-1})} \\
&E_tF_{t+1}(E_{t+1}F_{t+2})^{1-\delta_{t+1}}\dots(E_{T-2}F_{T-1})^{(1-\delta_{t+1})\dots(1-\delta_{T-2})} \\
&W_{t+1}(W_{t+2})^{1-\delta_{t+1}}\dots(W_{T-1})^{(1-\delta_{t+1})\dots(1-\delta_{T-2})} \\
&\delta_{t+1}^{-\delta_{t+1}}\delta_{t+2}^{-\delta_{t+2}(1-\delta_{t+1})}\dots\delta_{T-1}^{-\delta_{T-1}(1-\delta_{t+1})\dots(1-\delta_{T-2})}
\end{aligned}$$

The future  $EFs$  only include elements of the distribution of house prices for cases in which their corresponding future  $\lambda$  is positive. Using this equation together with the expression for  $\beta_t$  and the fact that

$$\begin{aligned} \frac{1}{1 + \delta_{t+1}(1 + r_{t+1})} &= 1 - \delta_t \\ \beta_t &= \left( \frac{\phi}{1 - \delta_t} \right)^{1-\delta_t} \delta_t^{-\delta_t} W_t (E_t \beta_{t+1})^{1-\delta_t} F_t \end{aligned}$$

gives equation (9)

### A.3 Proof of Proposition 2

The relevant first order conditions are

1. With refinancing ( $\lambda_t > 0$  and  $p_t - k - M_t \lambda_t > 0$ )

$$\begin{aligned} \exp(c_t^R) &= [\phi (E \beta_{t+1}) \delta_{t+1} (1 + r_{t+1})]^{-\frac{1}{b(1+\delta_{t+1}(1+r_{t+1}))}} \\ &\cdot \exp \{ \delta_t [(1 + r_t) A_t - M_t] \} \\ &\cdot W_t^{-\frac{1}{b(1+\delta_{t+1}(1+r_{t+1}))}} \exp [b \delta_t (\lambda_t p_t - k)] \end{aligned}$$

2. Without refinancing ( $\lambda_t > 0$  and  $p_t - k - M_t \lambda_t < 0$  or  $\lambda_t < 0$ ) we have:

$$\begin{aligned} \exp(c_t^{NR}) &= [\phi (E \beta_{t+1}) \delta_{t+1} (1 + r_{t+1})]^{-\frac{1}{b(1+\delta_{t+1}(1+r_{t+1}))}} \\ &\cdot \exp \{ \delta_t [(1 + r_t) A_t - M_t] \} \\ &\cdot W_t^{-\frac{1}{b[1+\delta_{t+1}(1+r_{t+1})]}} \exp (\delta_t \lambda_t M_t) \end{aligned}$$

### A.4 Proof of Proposition 3

There are 3 subcases to consider

#### A.4.1 Case 1

If  $\lambda_{t+1} > 0$  and  $p_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$ , then  $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$ . Hence, with certain house prices:

$$EF_{t+1}^c = F_{t+1}^c = \exp[-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k)]$$

as opposed to

$$EF_{t+1} = E \exp[-b\delta_{t+1}(p_{t+1}\lambda_{t+1} - k)]$$

From Jensen's inequality it follows that  $EF_{t+1} = E \exp(-x) > \exp(-Ex) = EF_{t+1}^c$  since  $\exp(-x)$  is convex.

$$c^C - c^U = \frac{1}{b[1 + \delta_{t+1}(1 + r_{t+1})]} \ln \left( \frac{EF_{t+1}}{EF_{t+1}^c} \right) > 0$$

where  $c^C$  is consumption when future house prices are certain and  $c^U$  is consumption under uncertainty. Here whilst certain future house prices are going to allow mortgage refinancing for sure, this may fail to be true with uncertain house prices. Therefore, house price uncertainty raises precautionary savings.

#### A.4.2 Case 2

If  $\lambda_{t+1} > 0$  and  $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$  then

$$EF_{t+1}^c = F_{t+1}^c = \exp[-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k)]$$

as opposed to

$$EF_{t+1} = \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})\gamma_t + E \left\{ \exp[-b\delta_{t+1}(p_{t+1}\lambda_{t+1} - k)] \left| p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}} \right. \right\} (1 - \gamma_t)$$

Since  $Ep_{t+1}\lambda_{t+1} - k > M_{t+1}\lambda_{t+1}$  we know that

$$\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1}) > \exp[-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k)]$$

However,  $E \left\{ \exp[-b\delta_{t+1}(p_{t+1}\lambda_{t+1} - k)] \left| p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}} \right. \right\}$  might be higher or lower than  $\exp[-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k)]$  depending on the nature of the house price distribution. Hence, the overall comparison is ambiguous: there is a risk that future house prices might be low, constraining the refinancing possibilities.

### A.4.3 Case 3

On the other hand if  $\lambda_{t+1} > 0$  and  $Ep_{t+1}\lambda_{t+1} - k < M_{t+1}\lambda_{t+1}$  then

$$EF_{t+1}^c = F_{t+1}^c = \exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1}) < \exp[-b\delta_{t+1}(\lambda_{t+1}Ep_{t+1} - k)]$$

Hence:

$$\frac{EF_{t+1}}{EF_{t+1}^c} = \gamma_t + \frac{E \left\{ \exp[-b\delta_{t+1}(p_{t+1}\lambda_{t+1} - k)] \left| p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}} \right. \right\}}{\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})} (1 - \gamma_t)$$

Since

$$\frac{E \left\{ \exp[-b\delta_{t+1}(p_{t+1}\lambda_{t+1} - k)] \left| p_{t+1} > M_{t+1} + \frac{k}{\lambda_{t+1}} \right. \right\}}{\exp(-b\delta_{t+1}\lambda_{t+1}M_{t+1})} < 1$$

then

$$\frac{EF_{t+1}}{EF_{t+1}^c} < 1$$

and

$$c^C - c^U = \frac{1}{b[1 + \delta_{t+1}(1 + r_{t+1})]} \ln \left( \frac{EF_{t+1}}{EF_{t+1}^c} \right) < 0$$

## A.5 Proof of Proposition 4

The effects of labour income risk: for the current period suppose that income were certain at the level  $\bar{w}_t = \alpha w_t + (1 - \alpha)B_t$ . Then since the exponential is a convex function and  $\delta > 0$

$$\begin{aligned} & \alpha \exp[-b\delta_{t+1}(1 + r_{t+1})w_t] + (1 - \alpha) \exp[-b\delta_{t+1}(1 + r_{t+1})B_t] \\ & < \exp[-b\delta_{t+1}(1 + r_{t+1})\bar{w}_t] \end{aligned}$$

and then since  $\ln(\cdot)$  is an increasing function, consumption is depressed by the labour income uncertainty. For future labour income risk the relevant terms are in  $E\beta_{t+1}$ ,  $W_{t+s}$ . If labour income of some future period were certain at its mean level this would increase the term in  $W_{t+s}$  which *ceteris paribus* would raise  $E\beta_{t+1}$  and tend to raise current consumption.

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