Trend Inflation, Wage Indexation, and Determinacy in the U.S. Technical Appendix

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Abstract

We describe the Christiano, Eichenbaum, and Evans (2005) model used to conduct our simulations.

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1 Households

Agents live forever and discount future at a rate β. There is a continuum of households that seek to maximize their expected utility function, given by:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(c_t - bc_{t-1}; h_t^s; m_t^h \right) \right\}. \tag{1}$$

E_0 defines the mathematical expectation operator conditional on the information set available at time 0. The function $u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right)$ is well-behaved and increasing in consumption c_t and money holdings m_t^h, decreasing in hours supplied h_t^s. Preferences display habit in consumption levels, measured by the parameter b.

There is a continuum of final goods, indexed by $i \in [0,1]$, that enter in the consumption bundle c_t through the usual Dixit-Stiglitz aggregator:

$$c_t = \left[\int_0^1 c_{it}^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where the parameter η indicates the elasticity of substitution between different varieties of goods. The standard household problem defines the optimal demand of good i, given by $c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t$, where P_t is the general price index given by $P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{\frac{1}{1-\eta}}$.

We assume a continuum of labour services h_{jt}, $j \in [0,1]$, that are combined according to the following technology

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} dj \right]^{\frac{\tilde{\eta}}{\tilde{\eta}-1}},$$

where $\tilde{\eta}$ is the elasticity of substitutions of labour types. Given production plans, a firm that minimizes costs has a labour-specific demand function given by $h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\eta}} h_t^d$, where W_{jt} is the wage prevailing in labour market j and W_t is a wage index defined as $W_t = \left[\int_0^1 W_{it}^{1-\tilde{\eta}} di\right]^{\frac{1}{1-\tilde{\eta}}}$. Integrating labour-specific demand functions one obtains h_t defined as

$$h_t \equiv \int_0^1 h_{jt} dj = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\eta}} dj. \tag{3}$$

Agents owns physical capital k_t that depreciates at rate δ. The capital accumulation equation is:

$$k_{t+1} = (1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right], \tag{4}$$

where the function S introduce a cost of varying the level of investment and satisfies the properties that $S(1) = S'(1) = 0$, $S''(1) > 0$.

Variable capacity utilization of physical capital is denoted by u_t, with an associated cost implicitly defined by $a(u_t)$. Agents owns firms and rent capital at a real interest rate r_t^k, earn profits and decide also over the utilization rate. Money is injected via lump-sum transfer τ_t. Finally the existence of complete markets on state contingent assets x_t^h assure that all agents choose the same level of consumption independently of the hours supplied. The budget constraint is then:

$$E_t r_{t,t+1} x_{t+1}^h + c_t + i_t + m_t^h + a(u_t) k_t = \frac{x_t^h}{\pi_t} + h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj + r_t^k u_t k_t + \phi_t - \tau_t \frac{m_{t-1}^h}{\pi_t} \quad (5)$$

Given wage stickiness à la Calvo wage dispersion $\int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$ can be written as:

$$w_t = \tilde{\alpha} w_{t-1} \frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} + (1-\tilde{\alpha}) \tilde{w}_t \quad (6)$$

where $\tilde{w}_t$ is the optimal wage set at time t.

The problem is to maximize (1) under eqs.(3)-(6). Household's first order conditions are hence given by

$$u_{c_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + u_{c_t}\left(c_{t+1} - bc_t; h_{t+1}^s; m_{t+1}^h\right) = \lambda_t \quad (7)$$

$$u_{h_t}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) = -\lambda_t \frac{w_t}{\tilde{\mu}_t} \quad (8)$$

$$q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[q_{t+1}(1-\delta) + r_{t+1}^k u_{t+1} - a(u_{t+1})\right] \quad (9)$$

$$q_t \lambda_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left[S_i\left(\frac{i_t}{i_{t-1}}\right)\right] i_t\right] - \beta q_{t+1} \lambda_{t+1} S_i \left(\frac{i_{t+1}}{i_t}\right) i_{t+1} = \lambda_t \quad (10)$$

$$a_{u_t}(u_t) = r_t^k \quad (11)$$

$$u_{m_t^h}\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) + \beta \frac{\lambda_{t+1}}{\pi_{t+1}} = \lambda_t. \quad (12)$$

Wages are sticky à la Calvo, and $1-\tilde{\alpha}$ is the probability of being able to reset wages next period. With probability $\tilde{\alpha}$ wages can not be re-optimized, and thus they are updated with past inflation, more precisely they vary according to $w_{j,t+1} = w_{j,t}\pi_t^{\tilde{\chi}}$ where $\tilde{\chi}$ is the degree of indexation to past inflation. Define $\tilde{w}_t$ as the optimal real wage set every period t. A union chooses the optimal wage maximizing its the utility function given by equation (1), subject to demand of labour in the specific market $h_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d$ and the probability of not being able to re-optimize in future periods. The resulting first order condition is:

$$E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}}\right] = 0. \quad (13)$$

Equation (13) states that optimal real wage must equate the future stream of marginal revenues from working to the expected sum of marginal cost of suppling labour. Given the structure of the model, all the reset optimal wages are identical in all labour markets in which the household can re-optimize. SGU shows how to write condition (13) in recursive form.

2 Firms

Each good is produced by a firm which monopolistically supplies its own variety using a production technology of the form:

$$z_t F(k_{it}, h_{it}) - \psi,$$

where z_t is an aggregate exogenous technology factor that follow an AR(1) process.ψ represent a fixed cost of production that generates increasing return to scale and guarantees zero profits in equilibrium. The production function $F(k_{it}, h_{it})$ is well-behaved and the same for all firms. Final goods can be used for consumption, investment, public expenditure and to pay cost of capital utilization. Each firm faces the following demand function:

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t, \tag{14}$$

where:

$$y_t = c_t + i_t + g_t + a(u_t) k_t. \tag{15}$$

We assume that firms can access a centralized market for capital, and must pay a fraction ν of wages at the beginning of the period by cash. Therefore their money demand function is:

$$m_{it}^f = \nu w_t h_{it} \tag{16}$$

Firms maximizes the expected value of future profits, under their demand function (14) and the cash-in-advance constraint (16). Firms' first order condition with respect to capital and labour services are:

$$mc_{it} z_t F_{k_{it}}(k_{it}, h_{it}) = r_t^k \tag{17}$$

$$mc_{it} z_t F_{h_{it}}(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{18}$$

If we assume that all firms have access to the same factor markets and F is homogeneous of degree one, equation (17) and equation (18) imply that all firms have the same marginal costs and aggregation across firms is straightforward.

Prices are sticky à la Calvo. Every period each firm can choose a new price of its own good with a probability $1-\alpha$. Those firms who can not reset their price update their price according to past inflation. Specifically their new price is $P_{it} = P_{it-1}\pi_{t-1}^{\chi}$ where χ is the degree of price indexation. The optimal price solve the first order condition:

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} y_{t+s} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\chi}}\right)^{\eta} \left[\frac{\eta-1}{\eta}\frac{\tilde{P}_t}{P_t} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{i,t+s}\right] = 0 \tag{19}$$

These expression states that optimizing firms choose a price $\tilde{P}_t$ that equates the expected sum of future marginal costs with the expected sum of marginal revenues, conditional on not being able to re-optimize in the future. Given the structure of the model, all the reset optimal prices are identical in all good markets in which firms can re-optimize. Schmitt-Grohe and Uribe (2004) (SGU henceforth) show how to write condition (19) in recursive form.

3 The Government

The government has two policy instruments: public expenditure and the nominal interest rate. Government expenditure is financed through lump-sum taxes and seigniorage:

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t}. \tag{20}$$

We assume an optimizing government that minimizes costs of acquiring the composite good, hence given public expenditure, government's absorption of a single variety of goods is $g_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} g_t$.

4 Equilibrium

Equilibrium on money market is simply: $m_t = m_t^h + m_t^f$. Equilibria in labour and capital markets imply that:

$$\int_0^1 h_{it}^d di = h_t^d \tag{21}$$

$$\int_0^1 k_{it} di = u_t k_t. \tag{22}$$

Consider equilibrium in the final goods' markets. The assumption that both government and agents minimize their expenditure choosing the optimal quantity of each variety of good implies the following condition:

$$z_t F(k_{it}, h_{it}) = (c_t + g_t + i + a(u_t) k_t) \left(\frac{P_{it}}{P_t}\right)^{-\eta}. \tag{23}$$

By integrating the right-hand side of the previous equation, considering that the capital-labour ratio is the same among firms, and imposing equations (21) and (22), we obtain:

$$z_t h_t^d F\left(\frac{u_t k_t}{h_t^d}, 1\right) = (c_t + g_t + i + a(u_t) k_t) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di, \tag{24}$$

where $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta}$ constitutes a wedge between aggregate supply and aggregate absorption and represents the price dispersion generated by price staggering. Then:

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{-\eta} s_{t-1} \tag{25}$$

and the equilibrium on final goods' markets is given by:

$$z_t F\left(u_t k_t, h_t^d\right) = (c_t + g_t + i + a(u_t) k_t) s_t. \tag{26}$$

Using the same properties, we aggregate equations (17) and (18) obtaining:

$$mc_t z_t F_{k_t}\left(u_t k_t, h_t^d\right) = r_t^k \tag{27}$$

$$mc_t z_t F_{h_t^d}\left(u_t k_t, h_t^d\right) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right]. \tag{28}$$

Finally, the expression for wage dispersion closely follows its price counterpart. Using labour-specific demand function we can write:

$$h_{jt}^d = \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} h_t^d \tag{29}$$

and integrating both sides and using equation (21) yields:

$$h_t = h_t^d \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj \tag{30}$$

Then defining $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t}\right)^{-\tilde{\eta}} dj$, it is easy to show that:

$$\tilde{s}_t = (1-\tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} \tilde{s}_{t-1} \tag{31}$$

Finally the definition of the price and the wage index generates a low of motion for the aggregate price and wage levels:

$$P_t^{1-\eta} = \alpha \left(P_{t-1} \pi_{t-1}^{\chi}\right)^{1-\eta} + (1-\alpha) \tilde{P}_t^{1-\eta} \tag{32}$$

$$w_t^{1-\tilde{\eta}} = \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\eta}} + (1-\tilde{\alpha}) \tilde{w}_t^{1-\tilde{\eta}} \tag{33}$$

5 Functional forms

As in SGU, we assume the following functional forms:

$$\begin{aligned}
u\left(c_t - bc_{t-1}; h_t^s; m_t^h\right) &= \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2} h_t^2 + \phi_1 \frac{\left(m_t^h\right)^{1-\sigma_m}}{1-\sigma_m} \\
F\left(u_t k_t, h_t^d\right) &= (u_t k_t)^\theta \left(h_t^d\right)^{1-\theta} \\
S\left(\frac{i_t}{i_{t-1}}\right) &= \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 \\
a(u_t) &= \gamma_1 (u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 .
\end{aligned}$$

The calibration is as in SGU and follows Christiano et al.'s (2005) estimation results. One period in the model is interpreted as a quarter. The steady state displays full capital utilization. Furthermore the quantities of money held by households and firms are set to match the empirical distribution,[1] likewise for long run inflation, 4.2% annual, and public expenditure, 52.44%. The rest of the calibration is listed in Table 1: the reader is referred to SGU and Christiano et al. (2005) for a discussion of exogenous processes and values use therein. The model is solved by employing the perturbation method developed in Schmitt-Grohe and Uribe (2004).

References

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Schmitt-Grohe, S., and M. Uribe (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28, 755–775.

[1] That is households hold 44% of money in steady state while fims hold the remaining 56%.