Appendix of "Calvo vs. Rotemberg in a Trend Inflation World: An Empirical Investigation" [not intended for publication]

Guido Ascari
University of Pavia and IfW

Efrem Castelnuovo
University of Padua and Bank of Finland

Lorenza Rossi
University of Pavia

May 2011

1 Estimation Procedure

To perform our Bayesian estimations we employed Dynare, a set of algorithms developed by Michel Juillard and collaborators (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2010)). Dynare is freely available at the following URL: http://www.dynare.org/. The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first $t \leq t_0 = 20,000$ draws. To do so, we computed the posterior mode by the "csminwel" algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix C_0 of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the "adaptive Metropolis" (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient that the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to "tune" the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix C_t of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, refer to Haario, Saksman, and Tamminen (2001).

We simulated two chains of 200,000 draws each, and discarded the first 80% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations.

2 Derivation of the alternative formulation of the Rotemberg model

We consider an alternative specification of the Rotemberg adjustment costs which introduces a wedge between production and labour inputs, as in the Calvo model. In particular, the intermediate good-producing firms i faces the following production function:

$$Y_t\left(i\right)=A_tN_t\left(i\right)-\frac{\varphi_p^N}{2}\left(\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}-1\right)^2Y_t$$

Rotemberg adjustment costs $\frac{\varphi_p^N}{2}\left(\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}-1\right)^2Y_t$ now result in an inefficiency loss in aggregate production. Formally, under a symmetric equilibrium it implies that:

$$N_t^d=\frac{Y_t}{A_t}\left(1+\frac{\varphi_p^N}{2}\left(\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}-1\right)^2\right)=\frac{Y_t\left(1+\Psi_t\right)}{A_t}. \tag{1}$$

Notice that similarly to the effect of price dispersion, $\Psi_t\geqslant 0$ implies that the higher the adjustment costs, the more labor N_t^d is needed to produce a given level of output. Therefore, as in the Calvo model, this way of modelling the adjustment costs creates a wedge between aggregate output and aggregate employment. In this case, the aggregate resource constraint is simply given by:

$$Y_t=C_t. \tag{2}$$

Total real costs are:

$$TC_t\left(i\right)=\frac{W_t}{P_t\left(i\right)}\left(i\right)N_t\left(i\right) \tag{3}$$

substituting (1) into equation (3), real total costs can be rewritten as

$$TC_t\left(i\right)=\frac{W_t\left(i\right)}{P_t\left(i\right)}\left(\frac{Y_t\left(i\right)+\frac{\varphi_p^N}{2}\left(\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}-1\right)^2Y_t}{A_t}\right) \tag{4}$$

and thus real marginal costs of firm i are

$$MC_t\left(i\right)=\frac{W_t\left(i\right)}{P_t\left(i\right)A_t} \tag{5}$$

The problem for the firm i is then:

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \left\{ \underbrace{\frac{P_{i,t+j}}{P_{t+j}} Y_{i,t+j}}_{\text{total revenues}} - TC_{i,t+j} \right\}, \tag{6}$$

$$\begin{aligned} \text{s.t. } Y_{i,t+j} &= \left[\frac{P_{i,t+j}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j}, & (7) \\ TC_{i,t+j} &= MC_{i,t+j} \left(Y_{i,t+j} + \frac{\varphi_p^N}{2} \left(\frac{P_{i,t+j}}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu} P_{i,t+j-1}} - 1 \right)^2 Y_{t+j} \right) & (8) \end{aligned}$$

where the notation is as above. Firms can change their price in each period, subject to the payment of the adjustment cost. Therefore, all the firms face the same problem, and thus will choose the same price and output. In other words the equilibrium is symmetric: $P_{i,t} = P_t, Y_{i,t} = Y_t$, $W_{i,t} = W_t$ and $MC_{i,t} = MC_t$ $\forall i$.

Substituting the constraint

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \left\{ \begin{array}{c} \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\varepsilon} Y_{t+j} - MC_{i,t+j} \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j} + \\ -MC_{i,t+j} \frac{\varphi_p^N}{2} \left(\frac{P_{i,t+j}}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu} P_{i,t+j-1}} - 1 \right)^2 Y_{t+j} \end{array} \right\} \tag{9}$$

First order condition implies at time t

$$\begin{aligned} 0 = & (1-\varepsilon) \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \frac{1}{P_t} Y_t + \varepsilon MC_t \left(\frac{P_{i,t}}{P_t} \right)^{-(\varepsilon+1)} \frac{Y_t}{P_t} + \\ & -MC_{i,t} \varphi_p^N \left(\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu} P_{i,t-1}} - 1 \right) \frac{1}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu} P_{i,t-1}} Y_t \\ & + \varphi_p^N E_t \left\{ \mathcal{D}_{t,t+1} MC_{i,t+1} \left(\frac{P_{i,t+1}}{\left(\pi_t^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu} P_{i,t}} - 1 \right) \frac{Y_{t+1} P_{i,t+1}}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu} P_{i,t}^2} \right\} \end{aligned} \tag{10}$$

imposing symmetry, dividing by $\frac{P_t}{Y_t}$ and rearranging:

$$\begin{aligned} (1-MC_t)\,\varepsilon = 1 \quad & -MC_t \varphi_p^N \left(\frac{\pi_t}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu}} - 1 \right) \frac{\pi_t}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu}} + \\ & + E_t \left\{ MC_{t+1} \beta \frac{\mathcal{U}'(C_{t+1})}{\mathcal{U}'(C_t)} \varphi_p^N \left(\frac{\pi_{t+1}}{\left(\pi_t^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu}} - 1 \right) \frac{\pi_{t+1}}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu}} \frac{Y_{t+1}}{Y_t} \right\} \end{aligned} \tag{11}$$

2.1 Log-linearization of the NKPC

The alternative NKPC in level is:

$$\begin{aligned}\underbrace{(1-MC_t)\,\varepsilon}_{\text{term 1}} &= \underbrace{1-MC_t\varphi_p^N\left(\frac{\pi_t}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}-1\right)\frac{\pi_t}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}}_{\text{term 2}}+ \\ &\quad \underbrace{+\varphi_p^N E_t\left\{MC_{t+1}\beta\frac{\mathcal{U}'(C_{t+1})}{\mathcal{U}'(C_t)}\left(\frac{\pi_{t+1}}{\left(\pi_t^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}-1\right)\frac{\pi_{t+1}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}\frac{Y_{t+1}}{Y_t}\right\}}_{\text{term 3}} \qquad (12)\end{aligned}$$

before log-linearizing consider its steady state value:

$$(1-MC)\,\varepsilon = 1-MC\varphi_p\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}+MC\beta\varphi_p\left(\pi^{1-\chi}-1\right)\pi^{1-\chi} \qquad (13)$$

Piece by piece log-linearization. The log-linearization of term 1 is

$$(1-MC_t)\,\varepsilon \simeq \varepsilon - MC\varepsilon - MC\varepsilon\widehat{mc}_t \qquad (14)$$

The log-linearization of term 2 is

$$1-MC_t\varphi_p^N\left(\frac{\pi_t}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}-1\right)\frac{\pi_t}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}} \simeq$$

$$\begin{aligned}\simeq\ & 1-MC\varphi_p\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}-\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}MC\widehat{mc}_t- \\ & -MC\varphi_p\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\hat{\pi}_t-\varphi_p^N MC\pi^{2(1-\chi)}\hat{\pi}_t+ \\ & +\varphi_p^N MC\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\chi\mu\hat{\pi}_{t-1}+\varphi_p^N MC\pi^{2(1-\chi)}\chi\mu\hat{\pi}_{t-1} \qquad (15)\end{aligned}$$

and finally the log-linearization of the term 3 is:

$$\varphi_p^N \beta E_t \left\{ \frac{Y_{t+1}^{-\sigma}}{Y_t^{-\sigma}} MC_{t+1} \left(\frac{\pi_{t+1}}{\left(\pi_t^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu}} - 1 \right) \frac{\pi_{t+1}}{\left(\pi_{t-1}^{\chi}\right)^{\mu} \left(\bar{\pi}^{\chi}\right)^{1-\mu}} \frac{Y_{t+1}}{Y_t} \right\} \simeq$$

$$\begin{aligned}
\approx \quad & MC\varphi_p\beta\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}+ \\
& +\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}MC\beta E_t\widehat{mc}_{t+1} \\
& +\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}MC\left(1-\sigma\right)\beta E_t\left(y_{t+1}-y_t\right) \\
& +\varphi_p^N MC\pi^{2(1-\chi)}\beta E_t\hat{\pi}_{t+1}+\varphi_p^N MC\pi^{1-\chi}\left(\pi^{1-\chi}-1\right)\beta E_t\hat{\pi}_{t+1}+ \\
& -\varphi_p^N MC\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\chi\mu\beta\hat{\pi}_t-\varphi_p^N MC\pi^{2(1-\chi)}\chi\mu\beta\hat{\pi}_t \qquad (16)
\end{aligned}$$

where we use the resource constraint equation which implies that $Y_t = C_t$.

Putting together the three log-linearized terms we get:

$$\begin{aligned}
\left(1-MC\right)\varepsilon-MC\varepsilon\widehat{mc}_t \quad = \quad & 1-MC\varphi_p\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}+MC\varphi_p\beta\left(\pi^{1-\chi}-1\right)\pi^{1-\chi} \\
& -\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}MC\widehat{mc}_t- \\
& -MC\varphi_p\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\hat{\pi}_t-\varphi_p^N MC\pi^{2(1-\chi)}\hat{\pi}_t+ \\
& +\varphi_p^N MC\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\chi\mu\pi_{t-1}+\varphi_p^N MC\pi^{2(1-\chi)}\chi\mu\pi_{t-1} \\
& +\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}MC\beta E_t\widehat{mc}_{t+1} \\
& +\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}MC\left(1-\sigma\right)\beta E_t\left(y_{t+1}-y_t\right) \\
& +\varphi_p^N MC\pi^{2(1-\chi)}\beta E_t\hat{\pi}_{t+1}+\varphi_p^N MC\pi^{1-\chi}\left(\pi^{1-\chi}-1\right)\beta E_t\hat{\pi}_{t+1}+ \\
& -\varphi_p^N MC\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\chi\mu\beta\hat{\pi}_t-\varphi_p^N MC\pi^{2(1-\chi)}\chi\mu\beta\hat{\pi}_t, \quad (17)
\end{aligned}$$

using the steady state (13), collecting terms, solving for $\hat{\pi}_t$ and simplifying we get:

$$\begin{aligned}
\hat{\pi}_t \quad = \quad & \frac{\mu\chi}{1+\beta\mu\chi}\hat{\pi}_{t-1}+\beta\frac{1}{1+\beta\mu\chi}E_t\hat{\pi}_{t+1}+\frac{\left(\pi^{1-\chi}-1\right)}{\left(1+\beta\mu\chi\right)\left(2\pi^{1-\chi}-1\right)}(1-\sigma)\beta E_t\left(y_{t+1}-y_t\right) \\
& +\frac{\left(\varepsilon-\varphi_p^N\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\right)}{\varphi_p^N\left(2\pi^{1-\chi}-1\right)\pi^{1-\chi}\left(1+\beta\mu\chi\right)}\widehat{mc}_t+\frac{\left(\pi^{1-\chi}-1\right)}{\left(1+\beta\mu\chi\right)\left(2\pi^{1-\chi}-1\right)}\beta E_t\widehat{mc}_{t+1} \qquad (18)
\end{aligned}$$

we can rewrite the NKPC as follows:

$$\hat{\pi}_t=\gamma_p\hat{\pi}_{t-1}+\gamma_f\beta\hat{\pi}_{t+1|t}+\gamma_{dy}\beta\left(1-\sigma\right)\Delta\hat{y}_{t+1|t}+\gamma_{mc}\widehat{mc}_t-\beta\gamma_{mc}'\widehat{mc}_{t+1}, \qquad (19)$$

where

$$
\begin{aligned}
\gamma_p &= \frac{\chi\mu}{(1+\beta\chi\mu)}, \\
\gamma_f &= \frac{1}{(1+\beta\chi\mu)} \\
\gamma_{dy} &= \frac{(\pi^{1-\chi}-1)}{(1+\beta\mu\chi)(2\pi^{1-\chi}-1)}, \\
\gamma_{mc} &= \frac{\left(\varepsilon-\varphi_p^N(\pi^{1-\chi}-1)\pi^{1-\chi}\right)}{\varphi_p^N(2\pi^{1-\chi}-1)\pi^{1-\chi}(1+\beta\mu\chi)}, \\
\gamma'_{mc} &= \frac{(\pi^{1-\chi}-1)}{(1+\beta\mu\chi)(2\pi^{1-\chi}-1)}.
\end{aligned}
$$

Notice that for $\pi = 1$ it holds that

$$
\begin{aligned}
\gamma_p &= \frac{\mu\chi}{(1+\beta\chi\mu)}, \\
\gamma_f &= \frac{1}{(1+\beta\chi\mu)} \\
\gamma_{dy} &= 0, \\
\gamma_{mc} &= \frac{\varepsilon}{\varphi_p^N(1+\beta\chi\mu)}, \\
\gamma'_{mc} &= 0.
\end{aligned}
$$

and that for $\pi = 1$ and $\mu = \chi = 0$, the NKPC becomes:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1|t} + \frac{\varepsilon}{\varphi_p^N}\widehat{mc}_t. \tag{20}$$

while in the standard Rotemberg model we have:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1|t} + \frac{\varepsilon-1}{\varphi_p^Y}\widehat{mc}_t \tag{21}$$

then, for the calibration strategy, it must hold that: $\frac{\varepsilon}{\varphi_p^N} = \frac{\varepsilon-1}{\varphi_p^Y}$, which implies:

$$\varphi_p^N = \frac{\varepsilon}{\varepsilon-1}\varphi_p^Y. \tag{22}$$

We now need to derive log-linear real marginal costs. Recall that real marginal costs are:

$$MC_t = \frac{W_t}{P_t A_t} \tag{23}$$

By substituting for the aggregate labor supply, we get

$$MC_t = \frac{C_t^{\sigma} N_t^{\varphi}}{A_t} \tag{24}$$

then in log-linear terms

$$\widehat{mc}_t = \sigma y_t + \varphi n_t - a_t. \tag{25}$$

From the aggregate production function we find

$$N_t = \frac{Y_t \left(1 + \Psi_t\right)}{A_t} \tag{26}$$

in steady state with $\Psi = \frac{\varphi_p^N}{2}\left(\pi^{1-\chi} - 1\right)^2$, and with $A = 1$,

$$\frac{Y\Psi}{AN} = 1 - \frac{Y}{N} = 1 - \frac{1}{1+\Psi} = \frac{\Psi}{1+\Psi} \tag{27}$$

From the log-linearization of the aggregate production function we get:

$$n_t = y_t + \frac{\Psi}{1+\Psi}\psi_t - a_t \tag{28}$$

and

$$\psi_t = \varphi_p^N \left(\pi^{1-\chi} - 1\right) \pi^{1-\chi} \left[\hat{\pi}_t - \chi\mu\hat{\pi}_{t-1}\right] \tag{29}$$

then real marginal costs can be rewritten as

$$\begin{aligned} \widehat{mc}_t &= \sigma y_t + \varphi\left(y_t + \frac{\Psi}{1+\Psi}\psi_t - a_t\right) - a_t \\ &= \left(\sigma + \varphi\right) y_t - \left(1 + \varphi\right) a_t + \frac{\varphi\Psi}{1+\Psi}\psi_t \\ &= \left(\sigma + \varphi\right) y_t - \left(1 + \varphi\right) a_t + \frac{\varphi\Psi}{1+\Psi}\varphi_p^N \left(\pi^{1-\chi} - 1\right) \pi^{1-\chi} \left[\hat{\pi}_t - \chi\mu\hat{\pi}_{t-1}\right] \end{aligned} \tag{30}$$

The IS curve is the standard one:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) \tag{31}$$

We can now write the reduced form solution, which is given by the following system of equations:

$$\hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + \gamma_f \beta \hat{\pi}_{t+1|t} + \gamma_{dy} \beta (1-\sigma) \Delta \hat{y}_{t+1|t} + \gamma_{mc} \widehat{mc}_t - \beta \gamma'_{mc} \widehat{mc}_{t+1}, \quad (32)$$

$$\widehat{mc}_t = (\sigma + \varphi) \hat{y}_t - (1+\varphi) a_t + \frac{\varphi \Psi}{1+\Psi} \varphi_p^N \left(\pi^{1-\chi} - 1 \right) \pi^{1-\chi} \left[\hat{\pi}_t - \chi \mu \hat{\pi}_{t-1} \right], \quad (33)$$

$$\hat{y}_t = \hat{y}_{t+1|t} - \sigma^{-1} \left(\hat{\imath}_t - \hat{\pi}_{t+1|t} \right) + g_t, \quad (34)$$

where

$$\begin{aligned}
\gamma_p &= \frac{\chi \mu}{(1+\beta \chi \mu)}, \\
\gamma_f &= \frac{1}{(1+\beta \chi \mu)} \\
\gamma_{dy} &= \frac{(\pi^{1-\chi} - 1)}{(1+\beta \mu \chi)(2\pi^{1-\chi} - 1)}, \\
\gamma_{mc} &= \frac{\left(\varepsilon - \varphi_p^N (\pi^{1-\chi} - 1) \pi^{1-\chi} \right)}{\varphi_p^N \pi^{1-\chi} (2\pi^{1-\chi} - 1)(1+\beta \mu \chi)}, \\
\gamma'_{mc} &= \frac{(\pi^{1-\chi} - 1)}{(1+\beta \mu \chi)(2\pi^{1-\chi} - 1)}, \\
\varphi_p^N &= \frac{\varepsilon}{\varepsilon - 1} \varphi_p^Y \\
\Psi &= \frac{\varphi_p^N}{2} \left(\pi^{1-\chi} - 1 \right)^2.
\end{aligned}$$

3 Further robustness checks

In comparing Calvo and Rotemberg, our empirical exercises support (i) models with trend inflation; (ii) the empirical superiority of the Calvo model, and (iii) the low (or zero) degree of indexation to past inflation called for by the Calvo model. These conclusions have been drawn by relying on some assumptions whose relevance for our findings deserves further scrutiny. Therefore, we perform some robustness checks along different relevant dimensions.

- *Estimation/different calibration of the trend inflation rate.* In our baseline exercises, we calibrate the trend inflation rate to inflation's sample mean. However, given that the magnitude of the trend inflation rate drives the relevance of the "extra-components" showing up in the NKPC (Calvo, Rotemberg) and the IS schedule (Rotemberg), as well as it exerts a non-linear impact on most of the parameters of the system, it is worth engaging in a sensitivity analysis along this dimension. First, we estimate the trend inflation rate as a free parameter. Our results turn out to be virtually unchanged with respect to the ones presented in the text - for brevity, we do not report them here. We then re-estimate the Calvo and Rotemberg models under two alternative trend inflation calibrations, i.e. 2% and 3%. Table 3 collects in columns second to fifth the results concerning our unrestricted estimates. Our main results are by and large robust to these perturbations. In particular, the Calvo model still fits the data better, and with a call for indexation lower than that by Rotemberg - notably, zero indexation belongs to the 90% credible set just in the Calvo cases. As regards the calibration of trend inflation, perhaps not surprisingly the marginal likelihoods tend to favor 2.5%, i.e. the annualized and percentualized inflation sample mean.

- *Informativeness of the prior on the indexation parameter.* Model comparison of nested models performed on the basis of improper priors (e.g. priors having infinite variance) may lead to biased results bases on an improper Bayes factor (Gelfand, 1996). In fact, our model comparison is based on diffuse but proper priors, which makes our model comparisons sensible. Of course, different priors may lead to different results because of their influence on the marginal likelihood. To verify the robustness of our results, we then re-estimate the baseline model by employing a different prior for our "key" indexation parameter. In particular, we assume $\chi \sim Beta(0.25, 0.10)$, a density with much more mass on indexation values in line with the literature (e.g. Smets and Wouters, 2007). Table 3 (last two columns) exhibits the results of this further check. The estimated indexation degrees are in this case somewhat closer, with Calvo suggesting 0.19 and Rotemberg 0.26. Still, the Calvo model is again favored by the data.

- *Drifts in trend inflation.* Our baseline exercises assume a constant trend inflation in the sample under investigation. Of course, even in a sample like the great moderation, drifts in the low-frequency component of the inflation rate may have occurred. To control for this aspect of the inflation rate, we re-estimate our models with Hodrick-Prescott filtered inflation, and focus on its cyclical component. This exercise is clearly a quick-fix, in that it does not allow us to consider the impact of trend inflation drifts on the convolutions of the NKPCs presented in Section 2. However, Cogley and Sbordone (2008) show that such impact is likely to be empirically negligible. Hence, this exercise is likely to control for the bulk of the effects stemming from the movements in trend inflation. Our findings turn out to be solid to the employment of this measure of cyclical inflation.

- *Piecewise quadratic trend.* Canova (1998) shows that different filters may induce dramatically heterogeneous representations of the business cycle. We then re-estimate our models with an alternative business cycle representation, which is obtained by detrending the log-real GDP with a quadratic trend. In detrending the series, as in the case of the Hodrick-Prescott filtering, we employ the extended sample 1954:IV-2008:II. In so doing, we account for the 1973:I break in the deterministic trend identified by Perron and Wada (2009), who show that differing filtering methods (Beveridge-Nelson, Unobserved Component) return the same picture of detrended output conditional on such a break.[1] Interestingly, our point estimates are similar to those obtained under Hodrick-Prescott filtering,

[1] We allow for both a break in the constant and in the slope coefficients.

thus confirming our benchmark results.

- *Frisch labor supply elasticity.* Our benchmark calibration is $\varphi = 1$. We experimented with a variety of different values belonging to the set $[0.5, 1.5]$, and verified that our results are clearly robust to these variations.[2]

Overall, our checks confirm our main results, i.e. trend inflation leads to a superior fit, and Calvo calls for a superior fit and a lower indexation degree with respect to Rotemberg.

References

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2010): "Dynare Manual, Version 4.1.1," http://www.dynare.org/documentation-and-support/manual.

Brooks, S., and A. Gelman (1998): "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, 7(4), 434–455.

Canova, F. (1998): "Detrending and Business Cycle Facts," *Journal of Monetary Economics*, 41, 475–512.

Cogley, T., and A. Sbordone (2008): "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *American Economic Review*, 98(5), 2101–2126.

Gelfand, A. E. (1996): "Model Determination Using Sampling-Based Methods," in W. R. Gilks, S. Richardson, and D. J. Spiegelhalter (eds.): Markov Chain Monte Carlo in Practice, London: Chapman and Hall.

Haario, H., E. Saksman, and J. Tamminen (2001): "An Adaptive Metropolis Algorithm," *Bernoulli*, 7(2), 223–242.

Perron, P., and T. Wada (2009): "Let's Take a Break: Trends and Cycles in US Real GDP," *Journal of Monetary Economics*, 56, 749–765.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycle: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

[2] The results of these three last robustness checks are available upon requests.

Param.	*Posterior Mean* [5th pct, 95th pct]					
	Calvo $\bar{\pi}=2\%$	*Rotemb.* $\bar{\pi}=2\%$	*Calvo* $\bar{\pi}=3\%$	*Rotemb.* $\bar{\pi}=2\%$	*Calvo* $\chi \sim Beta(1/4,1/10)$	*Rotemb.* $\chi \sim Beta(1/4,1/10)$
χ	0.15 [0.00,0.32]	0.37 [0.03,0.69]	0.13 [0.00,0.27]	0.44 [0.09,0.77]	0.19 [0.06,0.31]	0.26 [0.10,0.40]
θ	0.69 [0.60,0.78]	0.61 [0.50,0.72]	0.69 [0.61,0.78]	0.59 [0.48,0.69]	0.65 [0.57,0.75]	0.61 [0.52,0.70]
σ	2.44 [2.02,2.84]	2.34 [1.94,2.75]	2.46 [2.07,2.88]	2.31 [1.91,2.70]	2.37 [1.94,2.78]	2.25 [1.86,2.65]
ϕ_y	0.95 [0.89,1.00]	0.95 [0.89,1.00]	0.94 [0.88,1.00]	0.95 [0.90,1.00]	0.94 [0.89,1.00]	0.95 [0.90,1.00]
α_π	3.25 [2.67,3.83]	3.32 [2.74,3.85]	3.26 [2.69,3.83]	3.34 [2.78,3.87]	3.34 [2.77,3.90]	3.40 [2.87,3.94]
α_y	0.14 [0.06,0.23]	0.13 [0.05,0.21]	0.15 [0.06,0.23]	0.13 [0.05,0.21]	0.14 [0.05,0.22]	0.13 [0.05,0.21]
α_i	0.77 [0.71,0.84]	0.75 [0.68,0.83]	0.79 [0.72,0.85]	0.76 [0.70,0.83]	0.77 [0.71,0.83]	0.75 [0.68,0.82]
ρ_a	0.97 [0.95,0.99]	0.97 [0.95,0.99]	0.97 [0.94,0.99]	0.97 [0.95,0.99]	0.97 [0.95,0.99]	0.97 [0.95,0.99]
ρ_m	0.49 [0.39,0.59]	0.48 [0.38,0.59]	0.46 [0.35,0.57]	0.45 [0.34,0.56]	0.43 [0.32,0.55]	0.41 [0.30,0.52]
ρ_g	0.89 [0.84,0.94]	0.90 [0.86,0.95]	0.89 [0.84,0.94]	0.91 [0.86,0.95]	0.90 [0.85,0.95]	0.91 [0.86,0.95]
σ_a	0.0095 [0.0075,0.0113]	0.0090 [0.0072,0.0107]	0.0096 [0.0077,0.0115]	0.0090 [0.0072,0.0107]	0.0092 [0.0072,0.0109]	0.0087 [0.0070,0.0104]
σ_m	0.0020 [0.0015,0.0024]	0.0021 [0.0016,0.0026]	0.0018 [0.0014,0.0022]	0.0020 [0.0015,0.0025]	0.0019 [0.0015,0.0023]	0.0020 [0.0015,0.0025]
σ_g	0.0009 [0.0007,0.0011]	0.0008 [0.0007,0.0011]	0.0009 [0.0007,0.0011]	0.0008 [0.0007,0.0011]	0.0009 [0.0007,0.0011]	0.0009 [0.0007,0.0010]
Marg.Lik.	−37.92	−40.44	−35.29	−38.64	−32.58	−35.05

Table 1: **Posteriors for structural parameters.** 1 / 2 / 3 / 4: Models estimated under a 2 / 3 / 2.5 / 2 per cent trend inflation net rate (yearly rate, percentualized), full indexation to past / past / trend / past inflation, and a Beta(0.50,0.285) / Beta(0.50,0.285) / Beta(0.50,0.285) / Beta(0.25,0.10) prior density on the indexation parameter. The Table reports the posterior means and the [5th,95th] percentiles. The Marginal Likelihood are computed with the modified harmonic mean estimator by Geweke (1998). Details on the model estimation are reported in the text.