Monetary Policy Shocks and Cholesky-VARs: An Assessment for the Euro Area*

Efrem Castelnuovo
University of Padova

July 2011

Abstract

An estimated monetary policy VAR with 1993:IV-2008:III Euro data returns an insignificant response of inflation to a monetary policy shock identified with the widely-employed Cholesky restrictions. We replicate this evidence with a Monte Carlo exercise in which the true inflation reaction, according to an estimated DSGE model which we use as Data-Generating Process, is negative. Consequently, an insignificant reaction of inflation to a small-scale Cholesky-VAR monetary policy shock does not necessarily point to policymakers' inability to stabilize inflation.

JEL classification: C22, E47, E52.

Keywords: Cholesky monetary policy shocks, VARs, Dynamic Stochastic General Equilibrium models, Bayesian estimation, Euro Area.

*We thank Michal Andrle, Antti Ripatti, Fabio Canova, Martin Ellison, Peter Hammond, Eric Leeper, and participants at the Helsinki Center of Economic Research seminar series and the Royal Economic Society 2011 (Royal Holloway, London) annual conference for useful feedbacks on this and a closely related project. All remaining errors are ours. Author's details: Efrem Castelnuovo, Department of Economics, University of Padova, Via del Santo 33, I-35123 Padova (PD). E-mail: efrem.castelnuovo@unipd.it. Phone: +39 049 8274257. Fax: +39 049 8274211.

1 Introduction

The effects of a policy shock are often estimated by employing vector autoregressions. Figure 1 depicts the impulse responses obtained with a trivariate VAR for the Euro area considering measures of inflation, the output gap,[1] and the policy rate for the sample 1993:IV-2008:III. Our evidence points to the following 'facts':

- *the reaction of inflation is slightly positive and it is statistically insignificant* (at the 90% confidence level);
- *the reaction of the output gap is negative but borderline significant at best*;
- *the reaction of the policy rate is positive and significant for some quarters after the shock.*

This evidence is intriguing.[2] Monetary policy shocks are typically thought as being one of the 'weapons' possessed by policymakers to affect households' consumption decisions, aggregate demand, and prices. However, the evidence reported in Figure 1, which echoes that provided by Mojon (2008) with U.S. data, seems to suggest that such 'weapon' has a very limited power.[3] Or does it?

This paper shows that *a muted response of inflation* as the one in Figure 1 *is fully consistent with policy surprises 'significantly' affecting the inflation rate.* The reason is the following. The responses plotted in Figure 1 are obtained by assuming a Cholesky (recursive) economic structure. The nice feature of this strategy, popularized by Christiano, Eichenbaum, and Evans (1999) and commonly adopted in the literature, is that it does not require the researcher to take a stand on the identification of other shocks. As a key-identifying assumption, this structure imposes a delay to the reactions of 'slow

[1] Giordani (2004) shows that the estimated responses to a monetary policy shock are likely to be biased if a measure of potential output is omitted from the VAR. A detailed description of the data is postponed to Section 2.2.

[2] This result is hardly driven by the dimension of our VAR. Barigozzi, Conti, and Luciani (2010) estimate a Structural Dynamic Factor model on a panel of 237 Euro area quarterly series, sample 1980:I-2008:IV. They also find the response of inflation to a monetary policy shock identified with the standard Cholesky assumption to be insignificant. Intriguingly, they find evidence more favorable to the standard AD-AS interpretation of the effects of a monetary policy shock when adopting sign restrictions to identify such shock.

[3] This evidence is sample specific. When considering a longer sample including the 1970s, we found significant macroeconomic reactions - see our Appendix A. However, the price puzzle, i.e. a significantly *positive* reaction of inflation to an unexpected monetary policy tightening, occurs. Moreover, sources of instabilities such as the exchange rate turmoils occurred in the early 1990s may affect the outcome of moments computed with fixed-coefficients VARs. We then stick to the subsample 1993:VI-2008:III.

moving' variables such as inflation and output to a monetary policy shock. Differently, standard DSGE frameworks typically feature contemporaneous macroeconomic reactions to policy surprises.[4]

We show that this timing discrepancy is able to induce quantitatively important departures of the Cholesky-VARs (CVARs henceforth) responses from the 'true' ones. Interestingly, the CVAR responses obtained with our Monte Carlo exercise closely resemble those obtained with actual Euro area data. Therefore, the *muted reaction of inflation* shown in Figure 1 *may very well be an artifact due to the imposition of wrong 'zero' restrictions, more than a 'fact'.* In other words, a flat reaction of inflation to a monetary policy shock generated with Cholesky-VARs is fully consistent with an environment in which policymakers are able to affect inflation via policy surprises.

Our evidence relies upon VAR and DSGE frameworks estimated with aggregate data referring to the Euro area. Our sample begins in 1993:IV to cut off the exchange rate turmoils occurred in 1992-1993, which render the European monetary policy hard to model with policy rules featuring fixed coefficients (Gerlach and Schnabel (2000)). The end of the sample is justified by some recent evidence of a policy shift by the ECB after the bankruptcy of Lehman Brothers in September 2008 (Gerlach and Lewis (2011)). Part of our sample refers to a 'fictitious' European monetary policy. We interpret the observations before 1999 (year of the adoption of the Euro as an accounting currency) as referring to a weighted average of the country-specific policies adopted pre-1999, and our estimated reactions to a 'European monetary policy shock' as the average reaction across countries belonging to the Euro area.[5] While being clearly open to discussion, our choice

[4] There are notable exceptions in the literature - see Christiano, Eichenbaum, and Evans (2005), Boivin and Giannoni (2006), Altig, Christiano, Eichenbaum, and Lindé (2011), and Leitemo and Kilponen (2010).

[5] A related issue regards the degree of homogeneity of the estimated reactions to policy surprises across Euro area countries. Peersman and Smets (1999) find homogeneous reactions of a variety of macroeconomic indicators for the EU5, with the exception of Italy. Peersman (2004) obtains quite homogeneous output responses and a mild heterogeneity as for the inflation reaction. Peersman and Smets (2005) find heterogeneities at industry-level, and asymmetric reactions across different phases on the business cycle. Ciccarelli and Rebucci (2006) find that the cross-country differences in the EU4 group have not decreased during the run-up to EMU. Boivin, Giannoni, and Mojon (2008) detect heterogeneity in the responses of a variety of macroeconomic indicators. Barigozzi, Conti, and Luciani (2010) estimate a Structural Dynamic Factor model on a large panel of Euro area quarterly series and find evidence of country-specific responses to a monetary policy shock as for a variety of macroeconomic indicators. As pointed out in the paper, we interpret the reactions depicted in Figure 1 as a 'weighted average' across the Euro countries belonging to the Euro-11 group, which is the one we consider in our analysis. More importantly, the main point of the paper, i.e. the quantitative assessment of the discrepancy between the DSGE and CVARs impulse responses, is valid no matter what the degree of heterogeneity across Euro countries is.

may be rationalized with the common effort undertaken by several European countries to bring the inflation rate down to sustainable levels since the early 1980s. The use of synthetic European data is widespread among researchers (see e.g. Peersman and Smets (1999), Gerlach and Schnabel (2000), Smets and Wouters (2003), Andrés, López-Salido, and Vallés (2006), Surico (2007), Andrés, López-Salido, and Nelson (2009), Barigozzi, Conti, and Luciani (2010)).

A key-issue regards the calibration of the DSGE model employed in our Monte Carlo experiment. It is well known that the Cholesky identification scheme may be problematic if the Data Generating Process (DGP) of the economic series processed with the VAR is not recursive. Canova and Pina (2005) and Carlstrom, Fuerst, and Paustian (2009) discuss this issue at length, and use *calibrated* DGPs to illustrate the perils of using a recursive VAR. While being informative, an 'anything goes' issue arises in this kind of exercises, i.e. different calibrations of the same DSGE model may give rise to dramatically different assessments of the performance of recursive VARs. Figure 2 illustrates this point. Two alternative calibrations of the same DSGE model (which we present in the next Section) are employed; a set of pseudo-data concerning output, inflation, and the nominal interest rate is generated with either calibration; finally, recursive VARs are estimated with pseudo-data, and the impact of a monetary policy shock is assessed. The different impact of these two calibrations on the performance of our Cholesky-VARs is revealing. 'Calibration A', which is behind the results plotted in the left-hand column in Figure 2, would lead a researcher to judge the performance of the Cholesky-VAR as fully satisfactory. 'Calibration B', in contrast, induces a 'price puzzle', a mild reaction of output - if anything, an 'output puzzle', and a largely overstated policy rate reaction.[6] Evidently, the assessment of the ability of a Cholesky-VAR to recover the true effects of a structural monetary policy shock is an empirical issue. This is the reason why, in conducting our Monte Carlo analysis, it is imperative to employ an *estimated* DSGE model of the business cycle.

The paper develops as follows. Section 2 presents, estimates, and validates the standard new-Keynesian DSGE model with Euro area data we employ. Such model is employed as DGP in Section 3, which sets up our Monte Carlo experiment. In this Section we contrast the impulse responses generated with our estimated DSGE with those coming from the CVARs in a controlled environment. An interpretation

[6] The two calibrations are indicated at the bottom of Figure 2. Intriguingly, the only difference between the two calibrations refers to the degree of persistence of the technology shock - very persistent in the first calibration, white noise in the second. We investigate the role of this and other model's parameters in Section 4.

of our results, based both on some matrix-algebra on the DSGE-CVAR mapping as well as a battery of simulations, is provided in Section 4. Section 5 proposes a variety of robustness checks, which confirm the solidity of our main result. Section 6 notes connections with the literature. Section 7 summarizes our results.

2 A DSGE model as DGP

2.1 Model description

We work with a standard DSGE model (see e.g. King (2000), Carlstrom, Fuerst, and Paustian (2009)). The log-linearized version of the model is the following:

$$\begin{aligned}
\pi_t &= (1+\alpha\beta)^{-1}[\beta E_t\pi_{t+1} + \alpha\pi_{t-1} + \kappa y_t + \varepsilon_t^{\pi}], && (1)\\
y_t &= \gamma E_t y_{t+1} + (1-\gamma)y_{t-1} - \sigma^{-1}(R_t - E_t\pi_{t+1}) + Q(\rho_a - 1)a_t, && (2)\\
R_t &= (1-\tau_R)(\tau_\pi\pi_t + \tau_y y_t) + \tau_R R_{t-1} + \varepsilon_t^R, && (3)\\
x_t &= \rho_x x_{t-1} + u_t^x, u_t^x \sim N(0,\sigma_x^2), x \in \{\pi, a, R\}. && (4)
\end{aligned}$$

Eq. (1) is an expectational new-Keynesian Phillips curve (NKPC) in which π_t identifies the inflation rate, β the discount factor,[7] y_t the output gap, whose impact on current inflation is influenced by the slope-parameter κ, α the degree of indexation to past inflation, and ε_t^{π} the 'cost-push' shock; γ is the forward-looking component in the intertemporal IS curve (2); σ^{-1} is the households' intertemporal elasticity of substitution; the convolution $Q \equiv (1+\nu)(\sigma+\nu)^{-1}$ involves the inverse of the Frisch labor elasticity ν, and a_t identifies the technological shock; τ_π, τ_y, and τ_R are policy parameters in the Taylor rule (3); the monetary policy shock ε_t^R allows for a stochastic evolution of the policy rate. The model is closed with the AR(1) stochastic processes (4), which feature mutually independent white noise structural shocks.

Small scale models like the one presented here are currently employed in discussions involving academics and policymakers (Ellison (2010)). Andrés, López-Salido, and Vallés (2006) and Andrés, López-Salido, and Nelson (2009) successfully replicate the evolution of European macroeconomic aggregates with this or similar frameworks. A better identification of the forces driving the Euro area macroeconomic dynamics is

[7] In this class of new-Keynesian frameworks, the NKPC is not vertical in the long-run due to the presence of the discount factor $\beta < 1$. Our baseline calibration sets $\beta = 0.99$. We estimated the model with an alternative calibration, $\beta = 1$, which implies a vertical long-run NKPC. Our estimates turned out to be unaffected.

likely to be provided by a model à la Smets and Wouters (2003), which features a larger variety of shocks and frictions. On the other hand, the small-scale model we work with allows us to have a better control of the DSGE-CVAR mapping we focus on with our MonteCarlo experiment. Moreover, a recent paper by Herbst and Schorfheide (2010) shows that a small-scale model similar to the one employed in this paper performs at least as well as larger scale models along a variety of dimensions as for a variety of forecasting exercises.[8] Finally, Section 3.2 discusses the results of a comparison we conducted between the impulse responses to a monetary policy shock produced by our estimated DSGE model and those generated by a DSGE-VAR($\widehat{\lambda}$) model à la Del Negro, Schorfheide, Smets, and Wouters (2007).

2.2 Model estimation

We estimate the model (1)-(4) with Bayesian methods. We work with quarterly Euro area data, sample: 1993:IV-2008:III. The beginning of the sample is justified by our intention of excluding the period of exchange market turmoil in 1992-1993, which cannot be captured by Taylor rules (Gerlach and Schnabel (2000)). This choice is also corroborated by some empirical findings by Ciccarelli and Rebucci (2006), who find the long-run cumulative impact on output of a 'EMU policy shock' to be significantly lower after 1991. As for the end of the sample, we exclude the most recent observations to avoid dealing with the acceleration of the recent financial crises occurred with the bankruptcy of Lehman Brothers in September 2008. We estimate our model with three observables, i.e. inflation, a measure of the output gap, and a short-term interest rate.[9]

The vector $\boldsymbol{\xi} = [\beta, \upsilon, \kappa, \alpha, \gamma, \sigma, \tau_\pi, \tau_y, \tau_R, \rho_a, \rho_\pi, \rho_R, \sigma_a, \sigma_\pi, \sigma_R]^T$ collects the parame-

[8] More precisely, Herbst and Schorfheide (2010) contrast a small-scale new-Keynesian AD/AS model with a larger scale model à la Smets and Wouters (2007) in the context of forecasting exercises regarding the U.S. GDP, inflation, and the federal funds rate during the U.S. Great Moderation. They show that the two models attain very similar root-mean square errors. However, the Smets-Wouters model does not lead to a uniform improvement in the quality of the density forecasts and prediction of comovements. In particular, the predictive density for output appears to be poorly calibrated. Moreover, the small-scale model performs better in terms of predicting the sign of the deviations of output growth and inflation with respect to their targets.

[9] The source of the data is the OECD Main Economic Indicators. The output gap is computed as log-deviation of the real GDP with respect to the potential output estimated by the OECD with a production-function approach. The inflation rate is the quarterly growth rate of the GDP deflator. The short term rate refer to three month money market rates and treasury bill rates. The data refer to the Euro-11 aggregate, i.e. all the countries that joined the Euro area in 1999 with the exclusion of Luxembourg. For a detailed definition of these objects and the construction of the Euro area aggregates, see the 'Economic Outlook Database Inventory' available at http://www.oecd.org . Our series are demeaned prior to estimation. Details on our estimation are reported in Appendix B.

ters characterizing the model. We set $\beta = 0.99$ and $\nu = 1$.[10] The remaining priors are collected in Table 1. Details on the Bayesian algorithm are collected in our Appendix B.

Our posterior estimates are reported in Table 1. The parameters of the policy rule suggest an aggressive conduct to dampen inflation and output gap fluctuations. In particular, the reaction to inflation is basically the same as the one found by Andrés, López-Salido, and Vallés (2006). The reaction of output is higher than what typically found in the literature.[11] We also find an high degree of interest rate smoothing, a result in line with recent evidence on the 'Euro area Taylor rule' (Andrés, López-Salido, and Vallés (2006), Andrés, López-Salido, and Nelson (2009)). Policy shocks turn out to be serially correlated. Support for the joint modeling of endogenous and exogenous sources of the policy rate persistence is provided by Castelnuovo (2003) with U.S. data, and Fève, Matheron, and Poilly (2007) as for the Euro area. As for the remaining structural parameters, we find a fairly low degree of price indexation; a predominance of the 'backward looking' component of the IS curve, consistent with the presence of a substantial degree of habit formation in line with a finding in line with Andrés, López-Salido, and Vallés (2006) and Andrés, López-Salido, and Nelson (2009); autoregressive parameters of our shocks fairly low and all well below 0.9, a finding supporting the model's ability to replicate the dynamics of the European data.

Figure 2 (left column) plots our DSGE model estimates of the macroeconomic responses to a monetary policy shock. According to our estimated DSGE model, a policy surprise pushing the interest rate upwards opens a prolonged recession which induces a deflationary phase. The path of output is hump-shaped due to the endogenous persistence of the output process (Furher (2000)). Following the Taylor rule, policymakers control the policy rate to influence the real interest rate and drive the economy back to its steady state. This result, in line with conventional wisdom as for the effect of a policy-driven demand shock, clearly shows the role that policy surprises may play in influencing inflation.

[10] Further investigations performed by perturbing this baseline calibration confirmed the robustness of our results.

[11] This might be due to the different measure of output gap we employ, i.e. OECD output gap, which is somewhat different from the measure of Hodrick-Prescott filtered output often employed in the literature (the in-sample correlation between these two indicators of the business cycle is 0.80). Other reasons behind this difference may have to do with sample selection and estimation techniques.

2.3 Model validation

DSGE models are in general affected by misspecification (Del Negro, Schorfheide, Smets, and Wouters (2007)). However, misspecification is a relevant issue if it affects the moments of interest. Our paper is concerned with the estimation of the effects of a monetary policy shock. Del Negro, Schorfheide, Smets, and Wouters (2007) propose an assessment of the severity of DSGE models' misspecification based on a comparison between the moments estimated with a Bayesian VAR whose priors are calibrated by imposing the cross-equation restrictions implied by the DSGE model - called 'DSGE-VAR(∞)' - and the same moments estimated with a BVAR whose priors are calibrated by optimally relaxing such restrictions to maximize the value of the marginal likelihood function - called 'DSGE-VAR($\widehat{\lambda}$)'. Del Negro, Schorfheide, Smets, and Wouters (2007) show that a medium scale model like Smets and Wouters (2007), while being misspecified along some dimensions, is quite reliable as for impulse responses to monetary policy and technology shocks. When applying the Del Negro et al (2007) methodology to our case, we find that the small scale DSGE-VAR(∞) model returns impulse responses to a monetary policy shock which are qualitatively and quantitatively similar to those obtained with our best-fitting DSGE-VAR($\widehat{\lambda}$).[12] This result, obtained with synthetic Euro area data, is in line with the one put forward by Del Negro and Schorfheide (2004) with U.S. data. Therefore, our small scale model can be taken seriously as for the investigation of the macroeconomic effects of a monetary policy shock.

Is a recursive VAR able to recover the responses of our estimated DSGE model correctly? If not, are the distortions induced by the recursive assumption severe? We investigate these questions in the next Section.

3 DSGE vs. CVARs: A Monte Carlo experiment

In this Section we ask the following question:

> *Suppose our estimated DSGE model is the true DGP. Is a Cholesky-VAR able to correctly recover the effects exerted by the monetary policy shock u_t^R?*

We answer this question by contrasting the true (DSGE) impulse responses with those estimated with a CVAR in which the variables are ordered as follows: inflation,

[12] The hyper-parameter λ measures the degree of misspecification of the restrictions coming from the DSGE model, i.e. the lower λ, the higher the misspecification of the DSGE framework. Some details on the comparison between the impulse responses coming from our DSGE-VAR(∞) and DSGE-VAR($\widehat{\lambda}$) models are provided in our Appendix C.

output gap, policy rate. This ordering is standard, in that i) 'slow moving' variables are assumed to react with one lag to a monetary policy impulse, and ii) the monetary policy rate is assumed to immediately react to macroeconomic indicators.

Our Monte Carlo exercise works as follows. For $k = 1$ to K, we

- sample a realization of the vector $\boldsymbol{\xi}^k$ from the estimated posterior density $p(\boldsymbol{\xi} \mid \boldsymbol{Y})$, where $\boldsymbol{Y}$ is the set of observables employed to estimate our DSGE model;

- compute the DSGE model-consistent impulse responses conditional on $\boldsymbol{\xi}^k$, and store them;

- estimate the CVAR (ordering: inflation, output gap, nominal rate) impulse responses with artificial data $\boldsymbol{x}^k_{ps,[3:T]}$ generated with the DSGE model conditional on $\boldsymbol{\xi}^k$, and store them.

We run this algorithm by setting the number of repetitions $K = 5,000$, the horizon of the impulse response functions $H = 15$, and the length of the pseudo-data sample $T = 60$. The sample length coincides with that of the actual data sample (1993:IV-2008:III) we employ to estimate both our DSGE model and our CVAR whose impulse responses are plotted in Figure 1. Monetary policy shocks are normalized to induce an on-impact equilibrium reaction of the nominal rate equivalent to 25 quarterly basis points.

Figure 3 contrasts the impulse responses obtained with the DSGE model (left column) with those stemming from our CVARs (right column). Our CVARs return a wide range of inflation responses. On average, inflation takes negative values after the shock for about eight quarters before going back to its steady state. However, i) the uncertainty surrounding such response is very large; ii) the magnitude suggested by our CVARs is substantially smaller than that of the true (DSGE) reaction. Looking at our CVAR inflation response, one would be inclined to conclude that the effects of a monetary policy shock are mild and insignificant. In fact, they are substantial and significant.[13] Interestingly, the average inflation response generated with our Monte

[13] One must be careful in comparing different impulse responses in this kind of exercises. Figure 1 depicts classical, analytically computed 90% confidence intervals (bootstrapped 90% confidence intervals turned out to be quite similar). DGSE-related responses consider Bayesian credible sets conditional on the [5th,95th] percentiles. Differently, the CVAR responses computed in our MonteCarlo exercises are based on distributions constructed by considering 5,000 different realizations of the mean impulse responses conditional on VARs estimated on pseudo-data sampled from the joint density $p(\boldsymbol{\xi} \mid \boldsymbol{Y})$. However, comparisons involving differently computed impulse responses are informative as for the possible distortions due to the imposition of wrong identification schemes our our VARs.

Carlo CVARs is remarkably similar to the one obtained with our CVAR estimated with actual Euro area data (Figure 1). Consequently, *a muted response of inflation* to a monetary policy shock identified with standard Cholesky restrictions is *fully consistent* with policy surprises which do *affect* the macroeconomic environment.

Similar considerations can be done as for the reaction of output. Our CVARs underestimate the impact of the policy shock on output, and indicate a bordeline-significant reaction of our indicator of the business cycle when, in fact, the true reaction is clearly significant. Differently, and perhaps not surprisingly, the estimated path of the policy rate is extremely similar to the true one.

Wrapping up, our Monte Carlo exercise reveals that Cholesky-VARs have troubles in correctly recovering the effects of a monetary policy shock if the DGP is a standard DSGE monetary policy model of the business cycle. In particular, the reactions of inflation and the output gap are severely distorted. Differently, the reaction of the policy rate is correctly estimated. Interestingly, these Monte Carlo CVAR responses nicely line up with those obtained with actual Euro area data. Hence, one can state that a insignificant response of inflation estimated with a Cholesky VAR is fully consistent with a monetary policy shock truly exerting a negative impact on inflation.

4 Why do CVARs produce wrong responses?

Why do we get distorted responses with our CVARs? In particular, what is the role played be the assumption of recursiveness in our CVAR exercises? Let $\boldsymbol{z}_t = [\pi, a, R]^T$, $\boldsymbol{\varepsilon}_t = [\varepsilon_t^\pi, a_t, \varepsilon_t^R]^T$, $\boldsymbol{u}_t = [u_t^\pi, u_t^a, u_t^R]^T$, and consider the set of unique decision rules consistent with the rational expectation assumption and the structure of the DSGE model:[14]

$$\boldsymbol{z}_t = \boldsymbol{\Gamma} \boldsymbol{z}_{t-1} + \boldsymbol{B} \boldsymbol{\varepsilon}_t, \boldsymbol{\Gamma} \equiv \begin{bmatrix} a_1 & f_1 & e_1 \\ a_2 & f_2 & e_2 \\ a_3 & f_3 & e_3 \end{bmatrix}, \boldsymbol{B} \equiv \begin{bmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{bmatrix}, \tag{5}$$

where $\boldsymbol{\Gamma}$ and $\boldsymbol{B}$ collect convolutions of the structural parameters $\boldsymbol{\xi}$ of our DSGE model. Given that the third column of $\boldsymbol{B}$ does not display, in general, zeros, the monetary policy shock ε_t^R *immediately* affects *all* the variables of the system.

[14] This Section exploits some results obtained by Carlstrom, Fuerst, and Paustian (2009) under the constraints $\alpha = \gamma = 1$ and unitary variances. We assume the existence of a unique rational expectations equilibrium. For an investigation on the mapping relating DSGE and CVAR impulse responses in presence of multiple equilibria, see Castelnuovo and Surico (2010).

Under rational expectations, the system (1)-(4) has the following VAR(2) representation:

$$\boldsymbol{z}_t = \boldsymbol{A}_1 \boldsymbol{z}_{t-1} + \boldsymbol{A}_2 \boldsymbol{z}_{t-2} + \boldsymbol{B}\boldsymbol{u}_t, \tag{6}$$

where $\boldsymbol{A}_1 = \boldsymbol{\Gamma} + \boldsymbol{BFB}^{-1}$ and $\boldsymbol{A}_2 = -\boldsymbol{BFB}^{-1}\boldsymbol{\Gamma}$. The variance-covariance matrix of $\boldsymbol{Bu}$ is given by $\boldsymbol{B\Omega B}^T$, where $\boldsymbol{\Omega}$ is a diagonal matrix of full rank 3 with the variances of the shocks positioned on the main diagonal. For ease of exposition (and without loss of generality), we set $\boldsymbol{\Omega} = \boldsymbol{I}_3$.

When conducting an econometric exercise, the structural shocks $\boldsymbol{u}_t$ (see eq. 4) are not observable, and must be inferred. To do so, the econometrician can estimate a reduced form VAR(2)

$$\boldsymbol{z}_t = \boldsymbol{A}_1 \boldsymbol{z}_{t-1} + \boldsymbol{A}_2 \boldsymbol{z}_{t-2} + \boldsymbol{\zeta}_t,$$

where $\boldsymbol{\zeta}_t = [\zeta_t^\pi, \zeta_t^a, \zeta_t^R]^T$ is a vector of *residuals* whose variance-covariance $VCV(\boldsymbol{\zeta}) = \boldsymbol{\Lambda}$ is a full (non diagonal) $[3x3]$ matrix.

To recover the unobserved structural monetary policy shock u_t^R, a researcher must impose some restrictions on the structure of the VAR. A popular choice is to orthogonalize the residuals $\boldsymbol{\zeta}_t$ by imposing a Cholesky (recursive) structure to the system. This is consistent with the assumption of delayed effects of the 'monetary policy shock' on the variables ordered before the nominal interest rate in the vector $\boldsymbol{z}_t$. The unique lower triangular matrix $\widetilde{\boldsymbol{B}}$ is computed such that

$$\widetilde{\boldsymbol{B}}\boldsymbol{\varphi_t} = \boldsymbol{\zeta}, \text{ with } \widetilde{\boldsymbol{B}} = \begin{bmatrix} \widetilde{b}_1 & 0 & 0 \\ \widetilde{b}_2 & \widetilde{c}_2 & 0 \\ \widetilde{b}_3 & \widetilde{c}_3 & \widetilde{d}_3 \end{bmatrix}, \text{ and } \boldsymbol{\varphi_t} = \begin{bmatrix} \varphi_t^\pi \\ \varphi_t^a \\ \varphi_t^R \end{bmatrix}. \tag{7}$$

The Cholesky 'shocks' $\boldsymbol{\varphi_t}$, which are orthogonal and are assumed to have unitary variance, are then identified by computing the elements of the matrix $\widetilde{\boldsymbol{B}}$ such that

$$\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T = \boldsymbol{\Lambda}.$$

This implies that the equivalence $\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T = \boldsymbol{BB}^T$ must hold. Solving the system, it is then possible to express the elements of $\widetilde{\boldsymbol{B}}$ in terms of the objects belonging to $\boldsymbol{B}$.

Given the restriction

$$\widetilde{\boldsymbol{B}}\boldsymbol{\varphi}_t = \boldsymbol{B}\boldsymbol{u}_t \tag{8}$$

imposed by eqs. (6) and (7), one can relate the Cholesky-'shocks' $\boldsymbol{\varphi_t}$ to the DSGE structural shocks $\boldsymbol{u}_t$ and the elements belonging to the matrix $\boldsymbol{B}$ as follows:

$$\boldsymbol{\varphi}_t = \Phi \boldsymbol{u}_t = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} u_t^{\pi} \\ u_t^{a} \\ u_t^{R} \end{bmatrix}, \tag{9}$$

where $\Phi \equiv \widetilde{\boldsymbol{B}}^{-1}\boldsymbol{B}$. Consequently, the mapping going from the true DSGE shocks to the CVAR monetary policy 'shock' reads

$$\varphi_t^R = \phi_{31}u_t^{\pi} + \phi_{32}u_t^{a} + \phi_{33}u_t^{R}. \tag{10}$$

The Cholesky 'shock' φ_t^R is, in fact, a misspecified representation of the true monetary policy shock u_t^R. The standard Cholesky identification scheme recover the true policy shock only under the restrictions $\phi_{31} = \phi_{32} = 0$. These would occur under $d_1 = d_2 = 0$ in the monetary impulse vector $\boldsymbol{B}[:,3]$ in eq. (5), i.e. if the structural DSGE model would feature lags in the impact of the true monetary policy shock u_t^R on inflation and output. However, these restrictions are *not* consistent with the DSGE models employed by most researchers, the model we focus on in this paper included. The calibration conditional on our estimated posterior means implies the following values for the matrixes characterizing the set of decision rules (5):

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0.11 & 0.08 & -0.25 \\ -0.01 & 0.89 & -0.42 \\ 0.02 & 0.19 & 0.76 \end{bmatrix}, \text{ and } \boldsymbol{B} = \begin{bmatrix} 1.03 & -0.05 & -0.47 \\ -0.02 & -0.11 & -0.60 \\ 0.14 & -0.03 & 0.81 \end{bmatrix}.$$

Notice that $\boldsymbol{B}[1,3] = d_1 = -0.47$, and $\boldsymbol{B}[2,3] = d_2 = -0.60$. As a consequence, the Cholesky scheme *does* misspecify the monetary policy shock.

The mapping going from the structural parameters $\boldsymbol{\xi}$ to the elements of the $\boldsymbol{B}$ matrix is highly non-linear, and a closed form solution to express the latter as a function of the former is not available. However, one may resort to numerical approximation to assess to what extent the calibration of the DSGE model is responsible for the distortions affecting the VAR impulse responses. We then construct the empirical distributions of the ϕ_{3s}-coefficients and of their contribution to the volatility of φ_t^R by sampling 5,000 realizations of the structural parameters and exploiting the fact that $\Phi = \Phi(\boldsymbol{\xi})$ in (9).

Figure 4 plots the contributions of the structural shocks $\boldsymbol{u}_t$ to the volatility of the Cholesky shock φ_t^R. If the CVAR were able to correctly identify the monetary policy shock, the contributions of the cost-push shock and the technological shock to the volatility of the Cholesky-policy surprises would be zero. The contribution of the cost-push shock, indeed, turns out to be negligible (on average, about 2%). Differently, the participation of the technological shock is substantial - on average, about 23%. Interestingly, the weight ϕ_{32} of the technological shock in the linear combination (10) is negative (on average, -0.13). In contrast, the loading ϕ_{33} of the monetary policy shock is, in line with expectations, positive (on average, 3.33).

This evidence offers a rationale for the dampened CVAR-reactions of inflation and output as opposed to the true DSGE ones. A negative technology shock triggers a positive output gap, which exerts a positive pressure on inflation and the policy rate. At the same time, a monetary policy shock (a policy tightening) calls for a positive response of the policy rate, and negative reactions of inflation and the output gap. Therefore, the reduced form shock φ_t^R actually captures the *joint* effects of these two structural shocks, whose impacts on inflation and output partly offset each other, therefore leading to dampened reactions as those in our Monte Carlo exercise. Again, these reactions are very similar to those depicted in Figure 1. Wrapping up, the muted macroeconomic reactions depicted in Figure 1, and in particular the response of inflation, are likely to be an artifact due to the misspecification of the monetary policy shock, more than a true fact.

A note on the ability of CVARs to correctly recover the true monetary policy shock is warranted. Our result hinges upon our favorite 'calibration', i.e. our Bayesian estimates. However, following the discussion presented above, CVARs could perform better if the contribution of the technological shock to the volatility of φ_t^R were lower. To verify this hypothesis, we set $\rho_a = 0$ and rescale the draws of the standard deviation of the technology shock σ_a in our Monte Carlo exercise by dividing them by ten. As shown in Figure 5, the performance of CVARs improves substantially. The DSGE vs. CVARs reactions of output are indistinguishable (with the exception, by construction, of the on-impact reaction). The reactions of the policy rate are basically the same. The responses of inflation are still somewhat different, with the CVARs overstating the degree of uncertainty and somewhat downsizing the deflationary phase after the shock. All in all, our CVARs perform remarkably well in this 'weak technology shock' counterfactual scenario. This result is important, because it stresses that the magnitude of the distortions affecting the CVARs responses depend on the features of the DGP process. However, the unrestricted model is clearly favored by the data. The (log-) marginal likelihood of our baseline (unrestricted) DSGE model is -154.02; a restricted version of the model conditional on the constraint $\rho_a = 0$ is associated to a dramatically lower marginal likelihood, which reads -168.56 (both marginal likelihoods computed with Laplace approximation). Our most likely calibration points towards the substantial distortions of the CVAR-responses to a monetary policy shock.

5 Robustness checks

We check the robustness of our results along different dimensions.

- *Optimal selection of VAR lags.* The DSGE model has a finite VAR(2) representation in population. Then, the VARs in our Monte Carlo exercise are estimated with two lags. However, sample uncertainty may justify the use of a different number of lags for some draws $\boldsymbol{x}^k_{ps,[3:T]}$. We then re-run our exercise by allowing for an optimal number of lags selected according to the Schwarz criterion. Figure 6 plots the responses we obtained. The impact of sample uncertainty on optimal lag-selection is hardly noticeable.

- *DSGE model-consistent restrictions on the VAR coefficients.* We run an exercise in which we impose the DSGE model-consistent autoregressive matrixes $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ to our VARs. Figure 7 plots the outcome of this exercise, which confirms the clear discrepancy existing between the DSGE and the CVAR responses. Not surprisingly, the uncertainty surrounding our CVARs responses shrinks. However, the magnitude of both inflation and output responses is clearly lower than the true one.

- *Population moments.* Of course, sample uncertainty may be an issue. We then contrast population impulse responses in Figure 8. CVARs impulse responses are correct in sign. However, the magnitude of the responses of output and inflation is clearly dampened when compared to the true DSGE reactions.

- *'Euro' sample.* Hayo and Hofmann (2006) detect a difference in the reaction to the output gap between the Bundesbank and the European Central Bank. Boivin, Giannoni, and Mojon (2008) and Barigozzi, Conti, and Luciani (2010) point toward a possible break in the Euro area conditional correlations due to the adoption of the Euro. We then re-estimated our DSGE model with post-1999 data, and re-run our Monte Carlo exercise. Preliminary attempts to estimate our DSGE model with post-1999 turned out to be unfruitful due to some difficulties in achieving convergence to the ergodic posterior distribution. Some explorations suggested that such difficulty could be solved by fixing the parameter τ_π to the value 1.1. Conditional on this calibration, convergence was achieved smoothly (this set of estimates is available upon request). Figure 9 plots our impulse responses. Again, the Cholesky assumption turns out to be empirically relevant in that the CVAR responses deliver a quantitatively different message with respect to the true ones.

6 Connections with the literature

We note connections with some related literature. Fernández-Villaverde and Rubio-Ramírez (2006) and Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007) derive a necessary condition to ensure the 'invertibility' of the DSGE model, i.e. to ensure that a VAR can actually recover the structural shocks as modeled by the DSGE framework. Ravenna (2007) shows that truncated VARs may provide misleading indications when the true DGP is an infinite order VAR. Further investigations on the distortions coming from the truncation bias, mainly on the identification of the technology shock and the dynamic reaction of hours to it, are offered by Christiano, Eichenbaum, and Vigfusson (2006) and Chari, Kehoe, and McGrattan (2008). Benati and Surico (2009) show that estimated VARs may display heteroskedasticity in a world in which, by construction, the DSGE model assumed to be the data-generating process is homoskedastic but a policy break occurs. Poilly (2010) investigates the consequences of estimating a medium-scale DSGE framework featuring complementarity in consumption and real-balances with indirect inference based on impulse response matching. She shows that the estimated structural parameters are sensitive to the VAR identification scheme. In particular, a real balance effect is unveiled by the non-recursive VAR. Benati (2010) shows that counterfactuals based on CVAR models may deliver dramatically different indications as regards the role of systematic monetary policy with respect to those obtained with a DSGE model. Canova and Pina (2005) set up a Monte Carlo exercise in which they consider two calibrated DGPs (a limited participation model and a sticky price-sticky wage economy) to estimate a variety of short-run 'zero restrictions' VAR identification schemes. They find substantial differences between the predictions coming from the structural models and those implied by the estimated CVARs. Carlstrom, Fuerst, and Paustian (2009) propose a theoretical investigation on the consequences of the timing discrepancy between DSGE and CVARs as for the macroeconomic reactions to a monetary policy shock. They show that, a-priori, 'anything goes', i.e. conditional on given calibrations of the DSGE model, CVARs may return a variety of predictions, including price and output puzzles, responses in line with the true DSGE reactions, muted responses, and so on. In this paper we consider an invertible DSGE model that enjoys a finite order VAR(2) representation, i.e. no truncation bias is at work (in population). Our analysis provide empirical support to the simulations proposed by Canova and Pina (2005) and Carlstrom, Fuerst, and Paustian (2009), in that we deal with an *estimated* DSGE. Our focus is on monetary policy shocks, as opposed to systematic monetary policy. Our empirical investigation enables us to propose an interpretation of the facts documented in Figure 1 based on the timing discrepancy between DSGE and CVARs.[15]

7 Conclusions

A recursive VAR estimated with 1993:IV-2008:III Euro data returns an insignificant response of inflation to a monetary policy shock. This paper replicates this evidence by setting up a Monte Carlo exercise in which the true inflation reaction, estimated with a DSGE model which we use as data-generating process, is significantly negative. Our results suggest that i) Cholesky-VARs may produce substantially distorted inflation reactions to a monetary policy shock; ii) a flat reaction of inflation to a monetary policy shock generated with a Cholesky-VAR is fully consistent with an environment in which policymakers are able to affect inflation with policy surprises. This is due to the different timing assumptions of the standard DSGE vs. Cholesky-VAR frameworks. While the first one allows for an immediate impact of the policy surprises on inflation and output, the second one assumes that 'slow moving' variables react to such shocks with a lag. This subtle timing difference invalidates the shock identification based on the widely-employed Cholesky assumption. The Cholesky-VAR 'policy shock' is, in fact, a linear combination of the true structural shocks of the economy. In particular, we find that the technology shock significantly affects the macroeconomic reactions estimated with a Cholesky-VAR, and substantially distort them.

Our results suggest that impulse responses to policy surprises identified with Cholesky-VARs should be interpreted with great care.

[15] A somewhat related paper is Faust, Swanson, and Wright (2004). They show that zero responses to prices imposed by the CVAR methodology are not supported by the data when disturbances are inferred using futures data in a two step procedure. However, such paper deals with the issue of identification schemes within structural VARs - for which they do provide econometric testing - but it is silent on structural models.

Further material

Appendix A - VAR evidence, sample 1971:I-2008:III

We provide evidence concerning the effects of monetary policy shocks stemming from a trivariate VAR by working with a longer sample: 1970:I-2008:III. The output gap series is computed by applying the one-sided backward looking version of the Hodrick-Prescott filter (weight: 1,600) to the real GDP series (taken in logs and multiplied by 100).[16]

Figure 10 depicts the dynamics responses computed with the same Cholesky identification scheme applied to our baseline VAR (whose impulse responses are reported in Figure 1). Evidently, differences with respect to the baseline impulse responses arise. In particular, the VAR estimated with a longer sample suggests the presence of a persistent and significant price puzzle. The presence of such puzzle is unappealing, because it goes against the predictions of most standard models and can hardly be rationalized by any plausibly calibrated model embedding the cost-channel (Rabanal (2007), Castelnuovo (2011)). We then stick to the shorter 1993:IV-2008:III sample for our analysis.

Appendix B - Details on Bayesian Estimation

To perform our Bayesian estimation and model validation exercises (for the latter, see Appendix C), we employed `DYNARE`, a set of algorithms developed by Michel Juillard and collaborators (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011)). `DYNARE` is freely available at the following URL: http://www.dynare.org/.

The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first $t \leq t_0 = 20,000$ draws. To do so, we computed the posterior mode by the "csminwel" algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix $\boldsymbol{C}_0$ of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional

[16] We employed the 'one_sided_hp_filter_kalman.m' `MATLAB` code kindly provided by Alexander Meyer-Gohde (Humboldt University), which implements the procedure described on p. 301 of Stock, J.H. and M.W. Watson (1999), "Forecasting inflation," *Journal of Monetary Economics*, vol. 44(2), pages 293-335, October. The code is available at http://ideas.repec.org/c/dge/qmrbcd/181.html .

variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the "Adaptive Metropolis" (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient than the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to 'tune' the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix $\boldsymbol{C}_t$ of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, see Haario, Saksman, and Tamminen (2001).

We simulated two chains of 200,000 draws each, and discarded the first 90% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations. As it is common in this literature, we assumed the estimated parameters to be independently distributed.

Appendix C - Model validation with DSSW (2007)

Del Negro, Schorfheide, Smets, and Wouters (2007) elaborate on Del Negro and Schorfheide (2004) and construct a methodology to assess DSGE models' misspecification. In short, they estimate a Bayesian VAR whose priors are calibrated by appealing to the cross-equation restrictions of the DSGE model. The cross-equation restrictions are systematically relaxed to document how the model fit changes. The hyper-parameter regulating the degree of adherence to the cross equation restrictions is λ - the higher it is, the less misspecified the DSGE model is. Del Negro, Schorfheide, Smets, and Wouters (2007) estimate a DSGE-VAR(λ), i.e. the Bayesian VAR fitting the data at best. The parameter λ is estimated jointly with the remaining parameters of the model to maximize the value of the marginal likelihood function.[17] The DSGE-VAR($\widehat{\lambda}$) then represents the reference

[17] Del Negro, Schorfheide, Smets, and Wouters (2007) pick the value for λ which implies the highest

model against which the DSGE-VAR(∞) framework, whose VAR structure features the cross-equation restrictions coming from the DSGE framework. The comparison involves the moments of interest. Del Negro, Schorfheide, Smets, and Wouters (2007) compare impulse responses to identified shocks such as monetary policy shocks and technology shocks. Interestingly, while the DSGE model they consider (the medium scale DSGE model proposed by Smets and Wouters (2007)) is misspecified along some dimensions (i.e. $\widehat{\lambda} << \infty$), it turns out to be reliable as for the qualitative and quantitative effects of these structural shocks.

We estimate a DSGE-VAR(λ) conditional on our small-scale DSGE model. We conduct a preliminary analysis to assess the empirical fit of four different DSGE-VAR($\lambda\,|j\,$) models, $j \in \{1, 2, 3, 4\}$. The marginal likelihoods associated to these estimated models read -153.63, -127.91, -127.49, and -131.13, respectively. We then consider DSGE-VAR(2) and DSGE-VAR(3) models, and discard the remaining ones.

A key issue regards the identification of the structural shocks in a BVAR context. We know that, in general, the mapping linking reduced-form VAR residuals $\boldsymbol{\zeta}_t$ to structural shocks $\boldsymbol{u}_t$ reads $\boldsymbol{\zeta}_t = \widetilde{\boldsymbol{B}}\boldsymbol{\Omega}\boldsymbol{u}_t$, where $\widetilde{\boldsymbol{B}}$ is the Cholesky decomposition of the reduced-form residuals variance-covariance matrix $\boldsymbol{\Lambda}$, $\boldsymbol{\Omega}$ is an orthonormal rotation matrix, and the structural shocks are standardized to have unit variance, i.e. $E(\boldsymbol{u}_t\boldsymbol{u}_t^T) = I$. The initial impact of the structural shocks on the endogenous variables $\boldsymbol{z}_t$ in the VAR is given by

$$\left(\frac{\partial \boldsymbol{z}_t}{\partial \boldsymbol{u}_t^T}\right)_{VAR} = \widetilde{\boldsymbol{B}}\boldsymbol{\Omega}.$$

The data are silent as for $\boldsymbol{\Omega}$, in that $\boldsymbol{\Lambda} = \widetilde{\boldsymbol{B}}\boldsymbol{\Omega}\boldsymbol{\Omega}^T\widetilde{\boldsymbol{B}}^T = \widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T$, i.e. the likelihood function is not affected by the rotation matrix $\boldsymbol{\Omega}$. Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007) propose to construct a rotation matrix $\boldsymbol{\Omega}$ based on the DSGE model under scrutiny. In particular, the state space representation (6) suggests that the unique matrix $\boldsymbol{B}(\boldsymbol{\xi})$ is responsible for the contemporaneous effects of $\boldsymbol{u}_t$ on $\boldsymbol{z}_t$. A QR decomposition of $\boldsymbol{B}(\boldsymbol{\xi})$ returns the following:

$$\left(\frac{\partial \boldsymbol{z}_t}{\partial \boldsymbol{u}_t^T}\right)_{DSGE} = \boldsymbol{B}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{B}}^*(\boldsymbol{\xi})\boldsymbol{\Omega}^*(\boldsymbol{\xi})$$

where $\widetilde{\boldsymbol{B}}^*(\boldsymbol{\xi})$ is lower triangular and $\boldsymbol{\Omega}^*(\boldsymbol{\xi})$ is orthonormal. The strategy proposed

marginal likelihood out of a set of J DSGE-VAR(λ) models, with $\lambda \in \{\lambda_1, \lambda_2, ..., \lambda_J\}$ being different calibrations for the parameter λ. Differently, we estimate the parameter λ jointly with the remaining ones of the DSGE model.

by Del Negro and Shorfheide (2004) and Del Negro et al (2007), which we follow in this paper, is to use $\boldsymbol{\Omega}^*(\boldsymbol{\xi})$ as the rotation matrix to compute the DSGE-VAR impulse responses, i.e.

$$\left(\frac{\partial \boldsymbol{z}_t}{\partial \boldsymbol{u}_t^T}\right)_{DSGE-VAR} = \widetilde{\boldsymbol{B}}\boldsymbol{\Omega}^*(\boldsymbol{\xi}).$$

Figure 11 depicts the IRFs computed with our DSGE-VARs (BVARs). The left-hand side panel refers to those conditional on two lags. The estimated reactions of output and the policy rate are extremely similar. Qualitatively, also the reaction of inflation is very similar in the two models. In both models inflation moves in reaction of a policy shock, and does it persistently. Both reactions are hump-shaped. The DSGE model suggests a larger reaction of inflation than the data in the first three quarters. Turning to the right-hand side panel, which refers to the responses conditional on three lags, we verify the robustness of our results. If anything, things go even 'better' as for the reaction of inflation which, according to the data, is even more clearly negative in the short-run.

Overall, our DSGE model proves to be able to generate realistic predictions of the effects of unanticipated changes in monetary policy.

References

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2011): "Dynare: Reference Manual, Version 4," Dynare Working Paper No. 1, April.

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Lindé (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.

Andrés, J., J. López-Salido, and E. Nelson (2009): "Money and the Natural Rate of Interest: Structural Estimates for the United States and the Euro Area," *Journal of Economic Dynamics and Control*, 33, 758–776.

Andrés, J., J. López-Salido, and J. Vallés (2006): "Money in an Estimated Business Cycle Model of the Euro Area," *Economic Journal*, 116, 547–477.

Barigozzi, M., A. M. Conti, and M. Luciani (2010): "Measuring Euro Area Monetary Policy Transmission in a Structural Dynamic Factor Model," London School of Economics and Political Science, ECARES, and Università degli Studi di Roma "La Sapienza", mimeo.

Benati, L. (2010): "Are Policy Counterfactuals Based on Structural VARs Reliable?," European Central Bank Working Paper No. 1188.

BENATI, L., AND P. SURICO (2009): "VAR Analysis and the Great Moderation," *American Economic Review*, 99(4), 1636–1652.

BOIVIN, J., AND M. GIANNONI (2006): "Has Monetary Policy Become More Effective?," *Review of Economics and Statistics*, 88(3), 445–462.

BOIVIN, J., M. P. GIANNONI, AND B. MOJON (2008): "How Has the Euro Changed the Monetary Transmission Mechanism?," in: D. Acemoglu, K. Rogoff, and M. Woodford (eds.), NBER Macroeconomics Annual, Chicago: University of Chicago Press., 1–49.

BROOKS, S., AND A. GELMAN (1998): "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, 7(4), 434–455.

CANOVA, F., AND J. P. PINA (2005): "What VAR Tell Us About DSGE Models?," in Diebolt. C. and Kyrtsou C. (Eds): New Trends In Macroeconomics, Springer Verlag, New York, NY, 89-124.

CARLSTROM, C., T. FUERST, AND M. PAUSTIAN (2009): "Monetary Policy Shocks, Choleski Identification, and DNK Models," *Journal of Monetary Economics*, 56(7), 1014–1021.

CASTELNUOVO, E. (2003): "Taylor Rules, Omitted Variables, and Interest Rate Smoothing in the US," *Economics Letters*, 81(1), 55–59.

CASTELNUOVO, E. (2011): "Testing the Structural Interpretation of the Price Puzzle with a Cost Channel Model," *Oxford Bulletin of Economics and Statistics*, forthcoming.

CASTELNUOVO, E., AND P. SURICO (2010): "Monetary Policy Shifts, Inflation Expectations and the Price Puzzle," *Economic Journal*, 120(549), 1262–1283.

CHARI, V., P. J. KEHOE, AND E. R. MCGRATTAN (2008): "Are Structural VARs with Long-Run Restrictions Useful in Developing Business Cycle Theory?," *Journal of Monetary Economics*, 55, 1337–1352.

CHRISTIANO, L., M. EICHENBAUM, AND C. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

CHRISTIANO, L. J., M. EICHENBAUM, AND C. EVANS (1999): "Monetary Policy Shocks: What Have We Learned and to What End?," In: Taylor, J.B., Woodford, M. (Eds.): Handbook of Macroeconomics, Elsevier Science, 65–148.

CHRISTIANO, L. J., M. EICHENBAUM, AND R. VIGFUSSON (2006): *Assessing Structural VARs*. Daron Acemoglu, Ken Rogoff, and Michael Woodford (Eds.): NBER Macroeconomics Annual, MIT Press, Cambridge.

CICCARELLI, M., AND A. REBUCCI (2006): "Has the transmission mechanism of European monetary policy changed in the run-up to EMU?," *European Economic Review*, 50, 737–776.

DEL NEGRO, M., AND F. SCHORFHEIDE (2004): "Priors From General Equilibrium Models for VARs," *International Economic Review*, 45, 643–673.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007): "On the Fit of New-Keynesian Models," *Journal of Business and Economic Statistics*, 25(2), 124–162.

Ellison, M. (2010): "What have we learnt about inflation dynamics?," Discussion at Bank of England Roundtable, July.

Faust, J., E. T. Swanson, and J. H. Wright (2004): "Identifying VARs based on high frequency futures data," *Journal of Monetary Economics*, 51, 1107–1131.

Fernández-Villaverde, J., and J. F. Rubio-Ramírez (2006): "Economic and VAR Shocks: What Can Go Wrong?," *Journal of the European Economic Association*, 4(2-3), 466–474.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, T. J. Sargent, and M. W. Watson (2007): "ABCs (and Ds) of Understanding VARs," *American Economic Review*, 97(3), 1021–1026.

Furher, J. C. (2000): "Habit Formation in Consumption and its Implications for Monetary-Policy Models," *American Economic Review*, 90(3), 367–390.

Fève, P., J. Matheron, and C. Poilly (2007): "Monetary policy dynamics in the Euro area," *Economics Letters*, pp. 97–102.

Gerlach, S., and J. Lewis (2011): "ECB reaction functions and the crisis of 2008," CEPR Discussion Paper No. 8472.

Gerlach, S., and G. Schnabel (2000): "The Taylor rule and interest rates in the EMU area," *Economics Letters*, 67(2), 165–171.

Giordani, P. (2004): "An Alternative Explanation to the Price Puzzle," *Journal of Monetary Economics*, 51, 1271–1296.

Haario, H., E. Saksman, and J. Tamminen (2001): "An Adaptive Metropolis Algorithm," *Bernoulli*, 7(2), 223–242.

Hayo, B., and B. Hofmann (2006): "Comparing monetary policy reaction functions: ECB versus Bundesbank," *Empirical Economics*, 31(3), 645–662.

Herbst, E., and F. Schorfheide (2010): "Evaluating DSGE Model Forecasts of Comovements," University of Pennsylvania, mimeo.

King, R. G. (2000): "The New IS-LM Model: Language, Logic and Limits," *Federal Reserve Bank of Richmond Economic Quarterly*, 86(3), 45–103.

Leitemo, K., and J. Kilponen (2010): "Transmission Lags and Optimal Monetary Policy," *Journal of Economic Dynamics and Control*, forthcoming.

Mojon, B. (2008): "When Did Unsystematic Monetary Policy Have an Effect on Inflation?," *European Economic Review*, 52(3), 487–497.

Peersman, G. (2004): "The Transmission of Monetary Policy in the Euro Area: Are the Effects Different Across Countries?," *Oxford Bulletin of Economics and Statistics*, 66(3), 285–308.

Peersman, G., and F. Smets (1999): "The Taylor Rule: A Useful Monetary Policy Benchmark for the Euro Area?," *International Finance*, 2(1), 85–116.

——— (2005): "The Industry Effects of Monetary Policy in the Euro Area," *Economic Journal*, 115, 319–342.

Poilly, C. (2010): "Does money matter for the identification of monetary policy shocks: A DSGE perspective," *Journal of Economic Dynamics and Control*, 34, 2159–2178.

Rabanal, P. (2007): "Does Inflation Increase After a Monetary Policy Tightening? Answers Based on an Estimated DSGE Model," *Journal of Economic Dynamics and Control*, 31, 906–937.

Ravenna, F. (2007): "Vector Autoregressions and Reduced Form Representations of DSGE Models," *Journal of Monetary Economics*, 54, 2048–2064.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1, 1123–1175.

——— (2007): "Shocks and Frictions in US Business Cycle: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Surico, P. (2007): "The Monetary Policy of the European Central Bank," *Scandinavian Journal of Economics*, 109(1), 115–135.

Param.	*Interpretation*	*Priors*	*Posterior Means* [5h,95th]
β	Discount factor	*Calibrated*	0.99 [–]
v^{-1}	Frisch elasticity	*Calibrated*	1 [–]
κ	NKPC, slope	$Normal(0.1, 0.015)$	0.06 [0.03,0.09]
α	Price indexation	$Beta(0.5, 0.2)$	0.11 [0.02,0.20]
γ	IS, forw. look. degree	$Beta(0.5, 0.2)$	0.24 [0.08,0.39]
σ	Inverse of the IES	$Normal(3, 1)$	4.04 [2.67,5.41]
τ_π	T. Rule, inflation	$Normal(1.5, 0.3)$	1.18 [0.76,1.58]
τ_y	T. Rule, output gap	$Gamma(0.3, 0.2)$	1.71 [1.07,2.32]
τ_R	T. Rule, inertia	$Beta(0.5, 0.285)$	0.88 [0.84,0.92]
ρ_a	AR tech. shock	$Beta(0.5, 0.285)$	0.87 [0.78,0.95]
ρ_π	AR cost-push shock	$Beta(0.5, 0.285)$	0.07 [0.01,0.16]
ρ_R	AR mon. pol. shock	$Beta(0.5, 0.285)$	0.40 [0.18,0.63]
σ_a	Std. tech. shock	$InvGamma(1.5, 0.2)$	4.21 [2.59,5.86]
σ_π	Std. cost-push. shock	$InvGamma(0.35, 0.2)$	0.77 [0.61,0.91]
σ_R	Std. mon. pol. shock	$InvGamma(0.35, 0.2)$	0.29 [0.24,0.34]

Table 1: **Bayesian estimates of the DSGE model.** 1993:IV-2008:III Euro area data. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text.

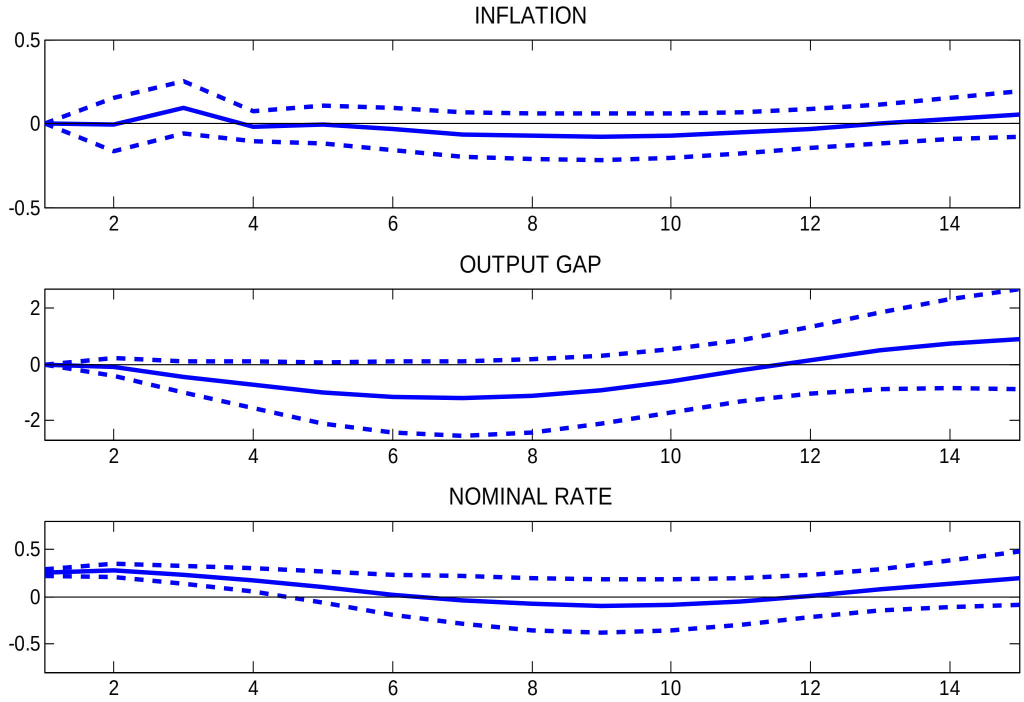

Figure 1: **CVAR impulse response functions to a monetary policy shock.** Sample: 1993:IV-2008:III. Variables: Quarterly GDP inflation, OECD output gap, quarterly short term interest rate - source: OECD Main Economic Indicators. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). Solid blue line: Mean response; dashed blue lines: 90% confidence interval (analytically computed). VAR estimated with a constant and three lags, optimally selected according to the Akaike information criterion. VAR estimated with a constant and three lags. Standard autocorrelation LM tests do not reject the null hypothesis of absence of serial correlation. The average Wald F-statistic associated to the Quandt-Andrews unknown breakpoint test conducted on each equation of the VAR does not reject the null hypothesis of no breakpoints within trimmed data (selected trimming: 20 per cent - test sample: 1996:IV-2005:III). Our results are robust to the employment of a different number or lags (either two or four).

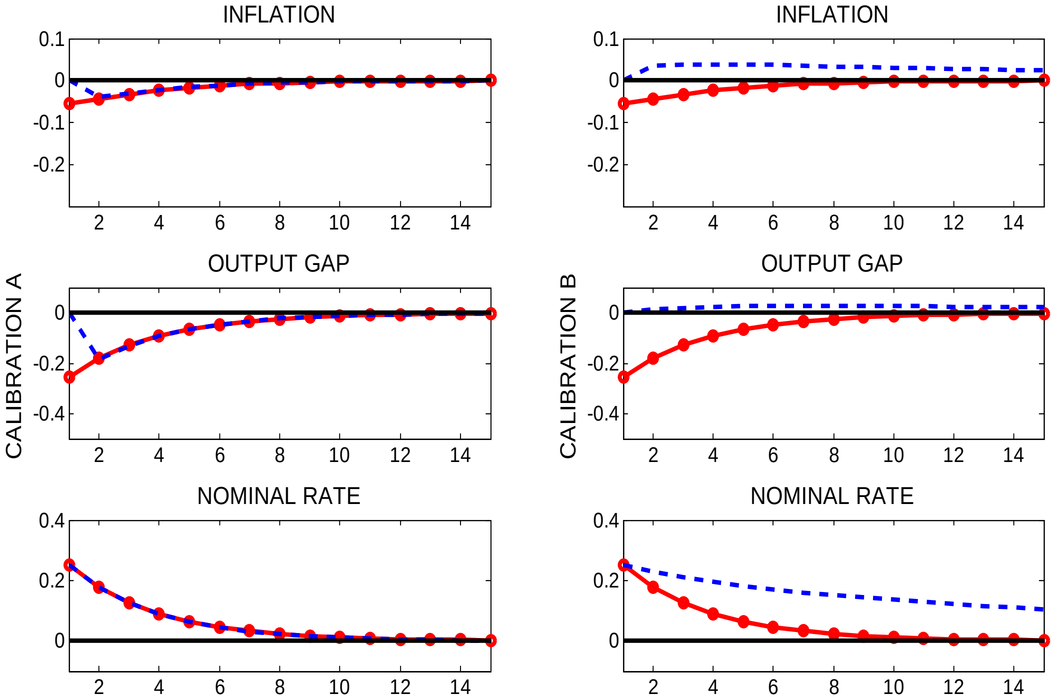

Figure 2: **DSGE and CVAR impulse response functions to a monetary policy shock - different calibrations.** Solid red lines: DSGE impulse responses; dashed blue lines: CVAR impulse responses (population moments). Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). VAR estimated with two lags. Model calibrations, common values: $\beta = 0.99$, $v = 1$, $\kappa = 0.06$, $\alpha = 0.11$, $\gamma = 1$, $\sigma = 4.04$, $\tau_\pi = 1.18$, $\tau_y = 1.72$, $\tau_R = 0.88$, $\rho_\pi = \rho_R = 0$, $\sigma_a = 4.21$, $\sigma_\pi = 0.77$, $\sigma_R = 0.29$; specific values, calibration A: $\rho_a = 0$; calibration B: $\rho_a = 0.95$.

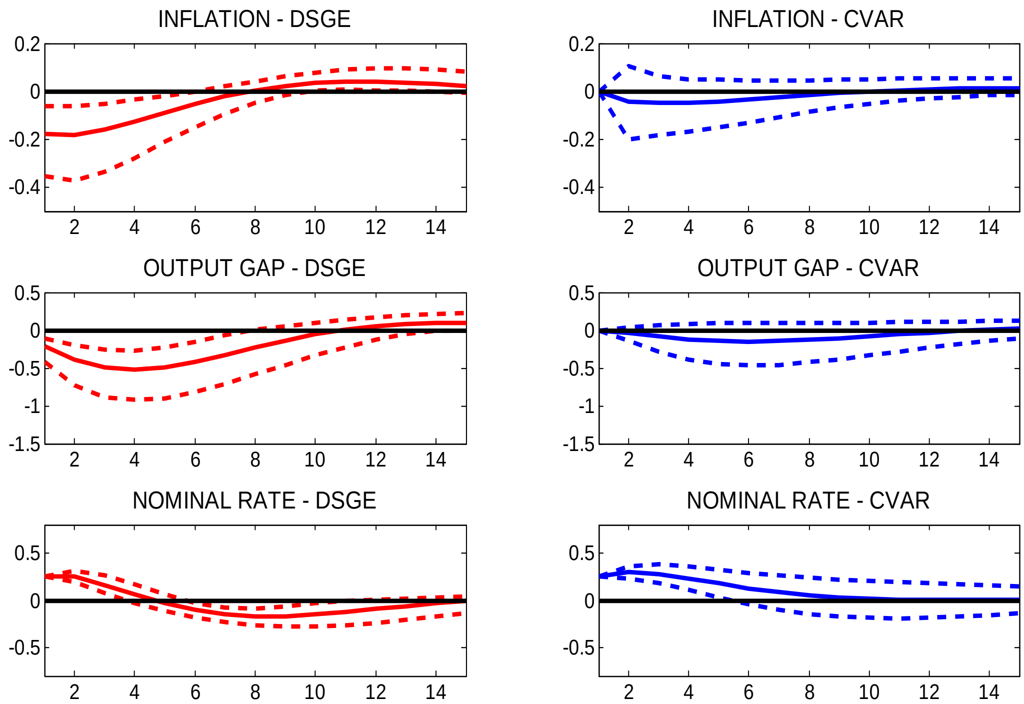

Figure 3: **DSGE and CVAR impulse response functions to a monetary policy shock.** Solid red lines: DSGE Bayesian mean impulse responses; dashed red lines: 90% credible sets. Solid blue lines: CVAR mean impulse responses; dashed blue lines: [5th,95th] percentiles; Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). VAR estimated with two lags.

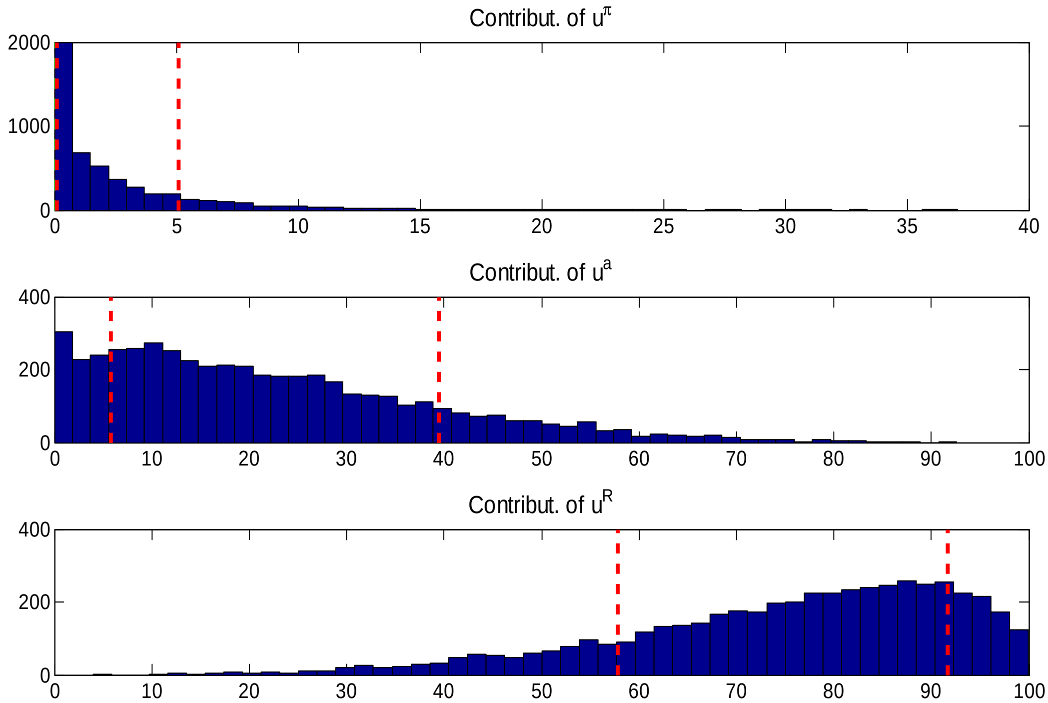

Figure 4: **CVAR monetary policy shock: Contributions of the true structural shocks.** Distribution computed over 5,000 stochastic simulations. Red dashed lines: [5th,95th] percentiles.

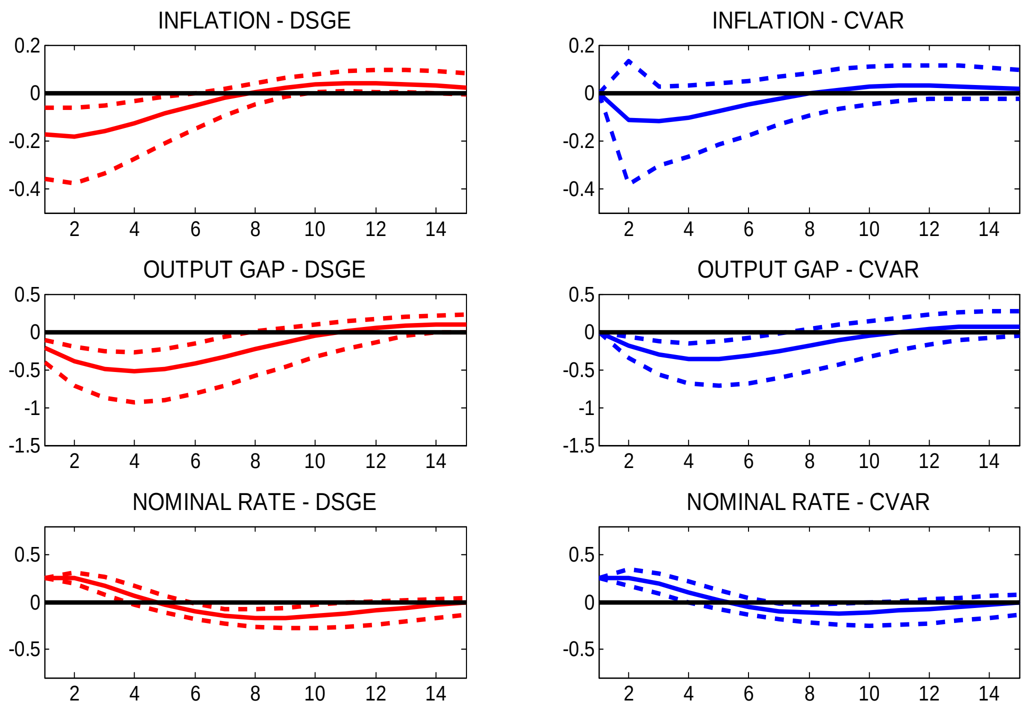

Figure 5: **DSGE and CVAR impulse response functions to a monetary policy shock - 'weakened' technology shock process.** Technology process featuring $\rho_a = 0$, and loaded by shocks drawn by a normal distribution whose variance is scaled down by a factor of ten. Solid red lines: DSGE Bayesian mean impulse responses; dashed red lines: 90% credible sets. Solid blue lines: CVAR mean impulse responses; dashed blue lines: [5th,95th] percentiles; Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). VAR estimated with two lags.

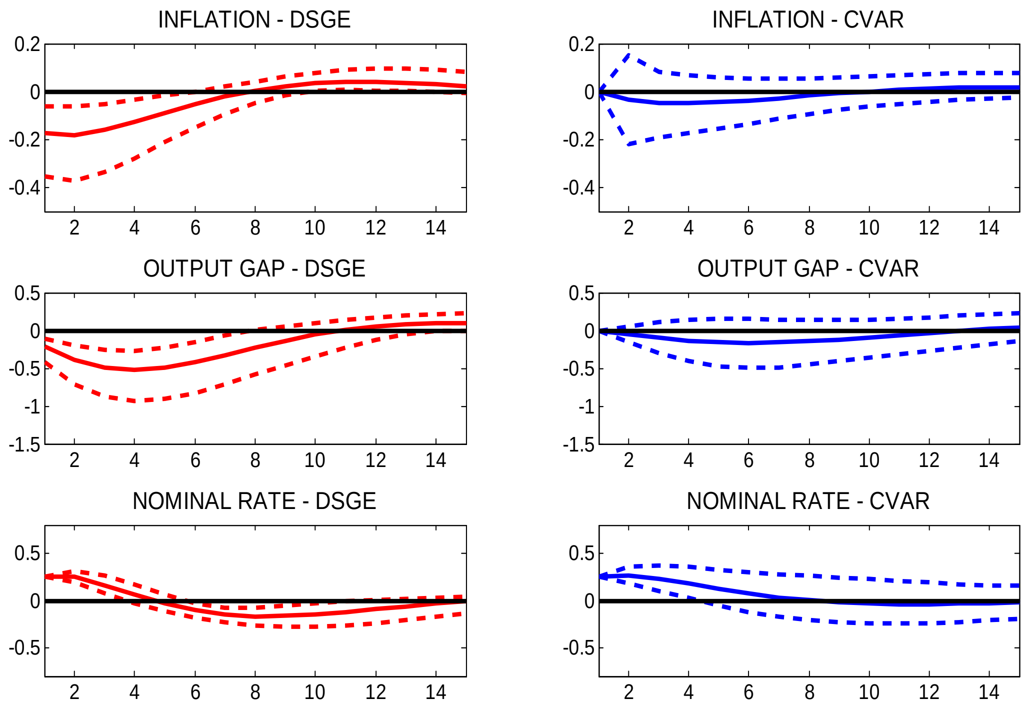

Figure 6: **DSGE and CVAR impulse response functions to a monetary policy shock - optimally selected VAR lags.** Solid red lines: DSGE Bayesian mean impulse responses; dashed red lines: 90% credible sets. Solid blue lines: CVAR mean impulse responses; dashed blue lines: [5th,95th] percentiles; Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate).

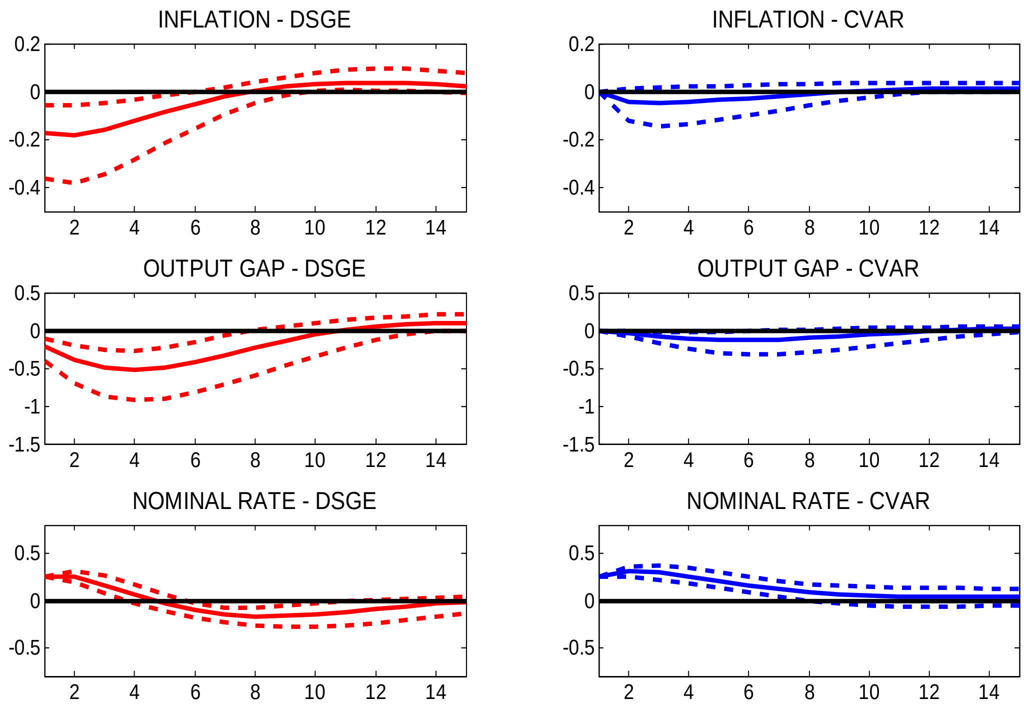

Figure 7: **DSGE and CVAR impulse response functions to a monetary policy shock - DSGE model-consistent restrictions imposed on the VAR coefficients.** Solid red lines: DSGE Bayesian mean impulse responses; dashed red lines: 90% credible sets. Solid blue lines: CVAR mean impulse responses; dashed blue lines: [5th,95th] percentiles; Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). VAR estimated with two lags.

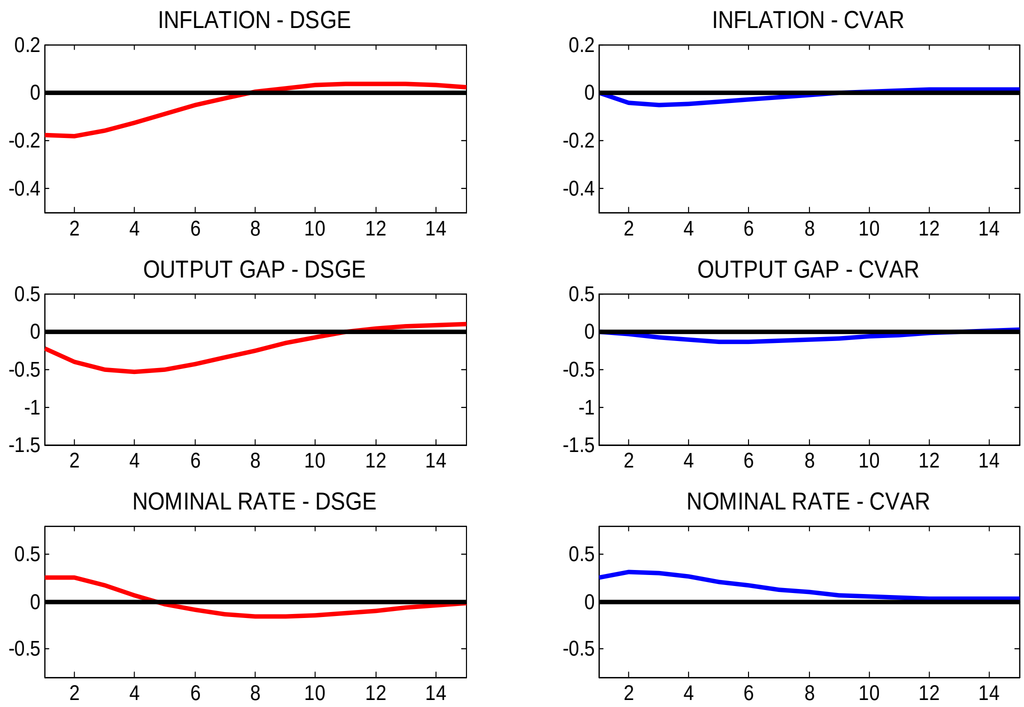

Figure 8: **DSGE and CVAR impulse response functions to a monetary policy shock - population moments.** Solid red lines: DSGE Bayesian mean impulse responses; dashed red lines: 90% credible sets. Solid blue lines: CVAR mean impulse responses; dashed blue lines: [5th,95th] percentiles; Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). VAR estimated with two lags.

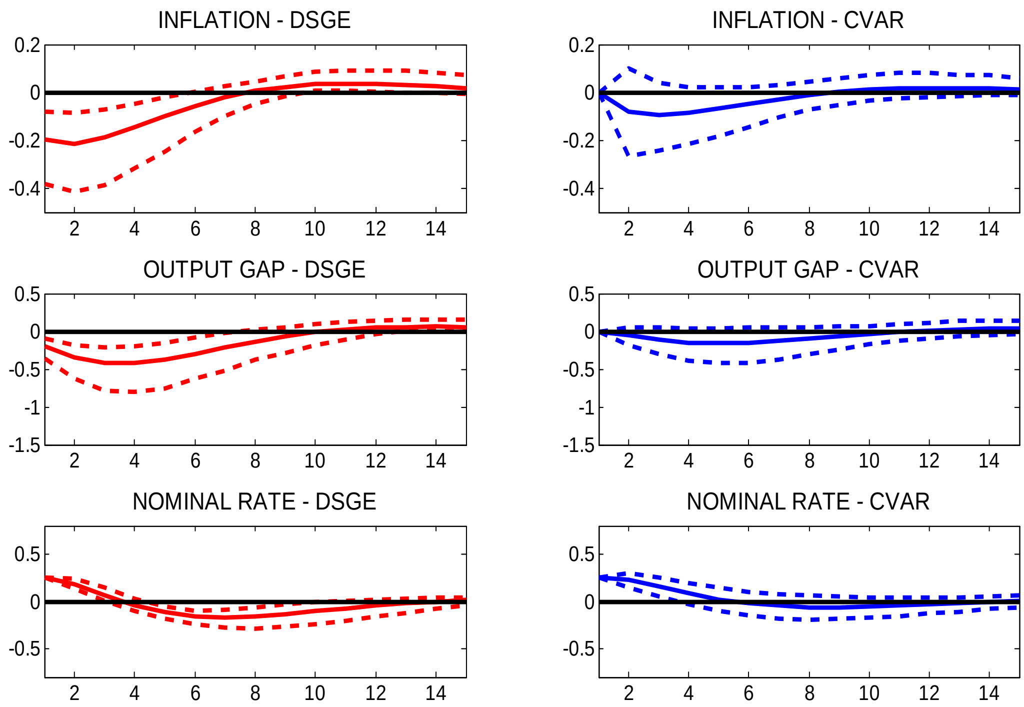

Figure 9: **DSGE and CVAR impulse response functions to a monetary policy shock - sample: 1999:I-2008:III.** Solid red lines: DSGE Bayesian mean impulse responses; dashed red lines: 90% credible sets. Solid blue lines: CVAR mean impulse responses; dashed blue lines: [5th,95th] percentiles; Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). VAR estimated with two lags.

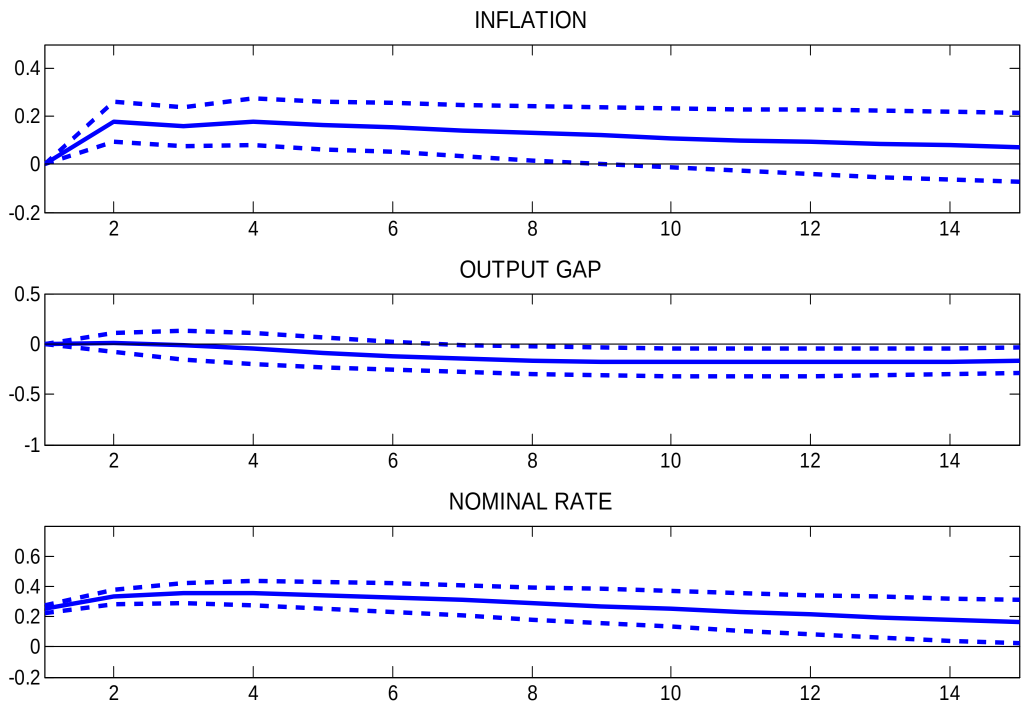

Figure 10: **CVAR impulse response functions to a monetary policy shock.** Sample: 1971:I-2008:III. Variables: Quarterly GDP inflation, output gap, quarterly short term interest rate - source: Euro Area Wide Model database. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output gap, short-term interest rate). Output gap computed by applying a backward looking Hodrick-Prescott filter (weight: 1,600) to the real GDP series (in logs and multiplied by 100). Solid blue line: Mean response; dashed blue lines: 90% confidence interval (analytically computed). VAR estimated with a constant and two lags, optimally selected according to the Schwarz criterion.

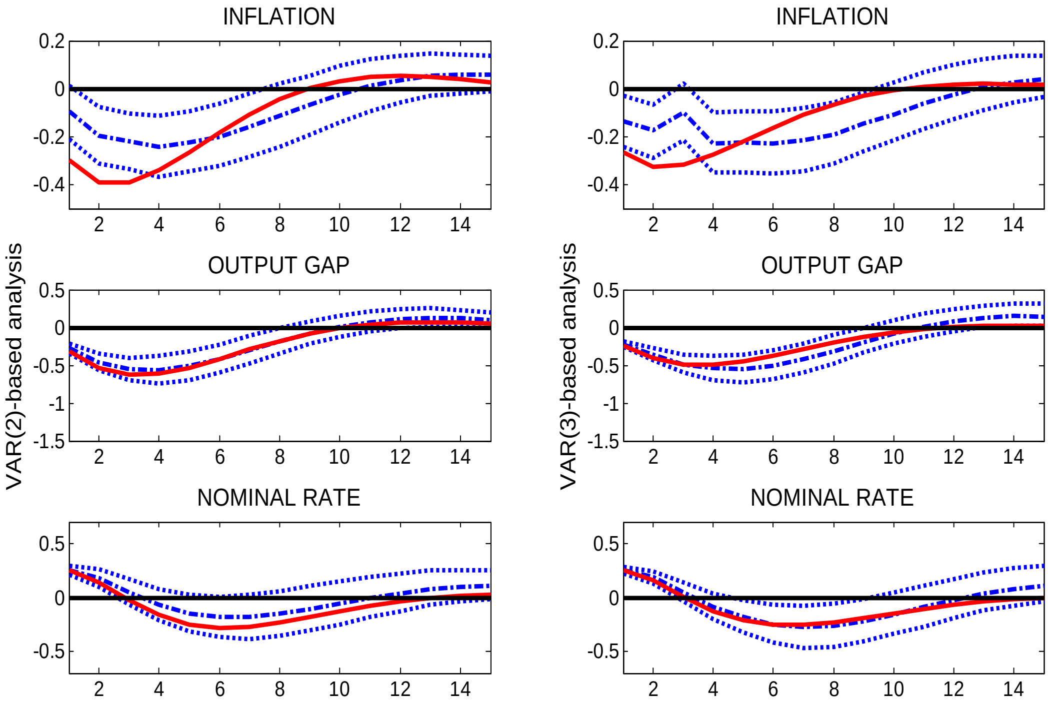

Figure 11: **DSGE-VAR($\widehat{\lambda}$) and DSGE-VAR(∞) impulse response functions to a monetary policy shock.** Dash-dotted (dotted) blue line (lines): Bayesian mean ([20th,80th] percentiles) impulse responses, DSGE-VAR($\widehat{\lambda}$). Solid red line: DSGE-VAR(∞) Bayesian mean impulse responses. Identification of the monetary policy shock as in Del Negro, Schorfheide, Smets, and Wouters (2007).