Appendix of "What does a monetary policy shock do? An international analysis with multiple filters" [not for publication]

Bayesian estimation

To perform our Bayesian estimations we employed `DYNARE`, a set of algorithms developed by Michel Juillard and collaborators. `DYNARE` is freely available at the URL http://www.dynare.org .

The mode of each parameter's posterior distribution was computed by using the 'csminwel' algorithm elaborated by Chris Sims. A check of the posterior mode, performed by plotting the posterior density for values around the computed mode for each estimated parameter in turn, confirmed the goodness of the optimizations. Such modes were employed to initialize the random walk Metropolis-Hastings algorithm for simulating the posterior distributions. The inverse of the Hessian of the posterior distribution evaluated at the posterior mode was used to define the variance-covariance matrix of the chain. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter. We simulated two chains of 200,000 draws each, and discarded the first 75% as burn-in. The variance-covariance matrix of the random walk chain was scaled to achieve an acceptance rate belonging to the [23%,40%] range for each estimated model. To assess the stationarity of the chains, we considered the convergence checks proposed by Brooks and Gelman (1998). We conditioned the estimation of the model to the unique-solution parameter region.

Posterior densities

Canova and Ferroni (2011) show that the MF approach is superior to single-filter alternatives in presence of sufficiently idiosyncratic information in the set of contaminated proxies of the business cycle employed in the estimation. The loadings of such proxies (posterior median values) range from 0.80 (LBR) to 4.88 (FD), therefore confirming the presence of heterogeneous information provided by the different filters. We then take the estimates obtained with MF as a reference when conducting comparisons across filters.

Table I collects our posterior densities of a subset of estimated parameters. To have a complete screening of our results, Figure I plots the densities of a subset of the estimated parameters of all our models. All the posterior medians appear to be economically sensible. Indeed, one may spot striking differences across filter-induced estimates. Remarkable filter-specific uncertainty surrounds the intertemporal elasticity of substitution, the extent to which agents are forward-looking in the IS curve, the long-run reaction of the Fed to inflation and output gap fluctuations, the persistence of the shocks, and their volatilities (with the exception of the volatility of the trend inflation shock, which appears to be fairly stable across filters). Given that policy counterfactuals are typically run by relying on such densities or, often, by conditioning on their means/medians, one may very well wonder how reliable the conclusions of such exercises should be considered in light of the just documented proxy-induced uncertainty.

It is worth scrutinizing how the model we focus on performs with respect to the standard price-indexation model displaying no trend inflation. To do so, we consider the version recently scrutinized by e.g. Benati and Surico (2008) and Benati (2008), which is featured by the following NKPC and Taylor rule:

$$\pi_t = \frac{\beta}{1+\alpha\beta}E_t\pi_{t+1} + \frac{\alpha}{1+\alpha\beta}\pi_{t-1} + \kappa x_t + \varepsilon_t^{\pi}, \tag{1}$$

$$R_t = (1-\phi_R)(\phi_\pi \pi_t + \phi_x x_t) + \phi_R R_{t-1} + \eta_t^R. \tag{2}$$

Eq. (1) displays the parameter α, which identifies non-reoptimizing firms' indexation to past inflation. Eq. (2) is a standard Taylor rule postulating a systematic reaction to inflation oscillations by the Fed. In a constant-inflation target world, inflation and inflation gap are coincident objects (up to a constant factor). However, as already mentioned, inflation target shocks have been empirically supported as one of the relevant driver of the post-WWII U.S. inflation (Cogley and Sargent (2005a), Ireland (2007), and

Cogley, Primiceri, and Sargent (2010)). If this is the case, a standard Taylor rule with a constant inflation target is likely to offer a misspecified representation of the U.S. monetary policy conduct.

We engage in a formal comparison between our benchmark (NKBC) model and the alternative indexation (IND) framework, which features a constant inflation target and equations (1) and (2). Following Benati (2008), we model the inflation shifter ε_t^π as a white noise, so giving the indexation parameter α the highest chance of grasping the U.S. inflation persistence. Table I collects the (log) Marginal Likelihoods of the NKBC and IND models (last two rows).[1] Three models out of five - HP, LIN, FD - support the price indexation model. However, the CBO filter and the MF panel of filters offers support to the benchmark NKBC model. Moreover, the NKBP model is naturally suited to investigate the role that trend inflation shocks have played for the post-WWII U.S. economy. Then, in the paper we focus on such a model.[2]

Forecast error variance decomposition

To gain some information on the role that filtering may play for the identification of the shocks driving the U.S. macroeconomic dynamics, we compute the model-consistent the forecast error variance decomposition at different horizons.[3] Table II collects percentage deviations of the filter-specific contributions of the two monetary policy shocks on inflation and output with respect to the one associated to MF. Once again, filtering matters. Some common patterns with the previously analyzed filter-specific impulse response functions arise. In fact, one may notice that HP, LIN, and FD suggest decompositions that are percentually very different with respect to the one proposed by MF, both when looking at 16-quarter ahead and when going for the 'long run' - 40-quarter ahead. Again, accounting for the break in the linear trend remarkably dampens the departures from MF, which are anyhow still present. Interestingly, while the standard monetary policy shock is subject to a very large amount of filter-induced uncertainty,

[1] Preliminary attempts to estimate the IND model with the priors reported in Table 2 failed due to the difficulty of computing posterior modes of the models at hand. We verified that a smooth convergence was instead possible by manipulating the prior mean of the slope of the NKPC. The estimations of the IND model are then conditional on $\kappa \sim Gamma(0.035, 0.01)$.

[2] Notice that we *cannot* discriminate across filters on the basis of the Marginal Likelihood. This is due to the procedure at hand, which implies a different data set for each estimated model. Differently, **?** filters the raw data and estimates the DSGE cyclical model *jointly*, i.e. in a single-step fashion. Therefore, his methodology allows for model comparisons based on the Marginal Likelihood even in presence of different filters.

[3] We thank Marco Ratto for kindly providing us with the 'vardec.m' code to compute this statistic.

that surrounding the contribution of trend inflation shocks for the inflation process is much lower. Indeed, the highest departure concerning this latter shock is that of LIN - about 35%. Much smaller figures are those associated to the reactions to an unexpected inflation target hike. This suggests that the large contribution assigned to trend inflation shocks by all filters as regards inflation is a very robust fact. This finding lines up with recent research -Ireland (2007), Cogley and Sbordone (2008), and Cogley, Primiceri, and Sargent (2010) pointing towards trend inflation shocks as the main inflation driver of the post-WWII U.S. period.

Wrapping up, the aforementioned evidence documents a marked filter-induced heterogeneity in the posterior densities and, consequently, in the estimated dynamic responses and variance decompositions as for the U.S. macroeconomic environment.

Impulse Responses - Multiple Filters Model, Robustness to the Exclusion of Selected Indicators

As already commented in the main text, we also undertook the estimation of our DSGE model with Multiple Filters by eliminating either the 'FD' indicator or the 'BP' one. Figures II and III depict the outcome of these exercises, which confirms the substantial indicator-induced heterogeneity affecting the impulse responses we focus on. We also experimented with a combination of the six multiple filters 'MF' with not-normalized weights. Figure IV depicts the outcome of our empirical exercise. With respect to the benchmark case depicted in Figure 2 (main text), the reactions of output to both shocks are quantitatively magnified. Obviously, this implies an increase of the percent deviations of the single-filter reactions with respect to the reaction associated to the 'MF' approach. Qualitatively similar, but quantitatively more moderate is the variation of the responses of the nominal interest rate. Differently, the reaction of inflation conditional on the MF filter constructed with not-normalized weights is slightly milder. Our main result, i.e., the large heterogeneity implied by the use of different filters in the estimation of DSGE models and the implied impulse responses to monetary policy shocks, is fully confirmed by our analysis with MF computed with not-normalized weights.

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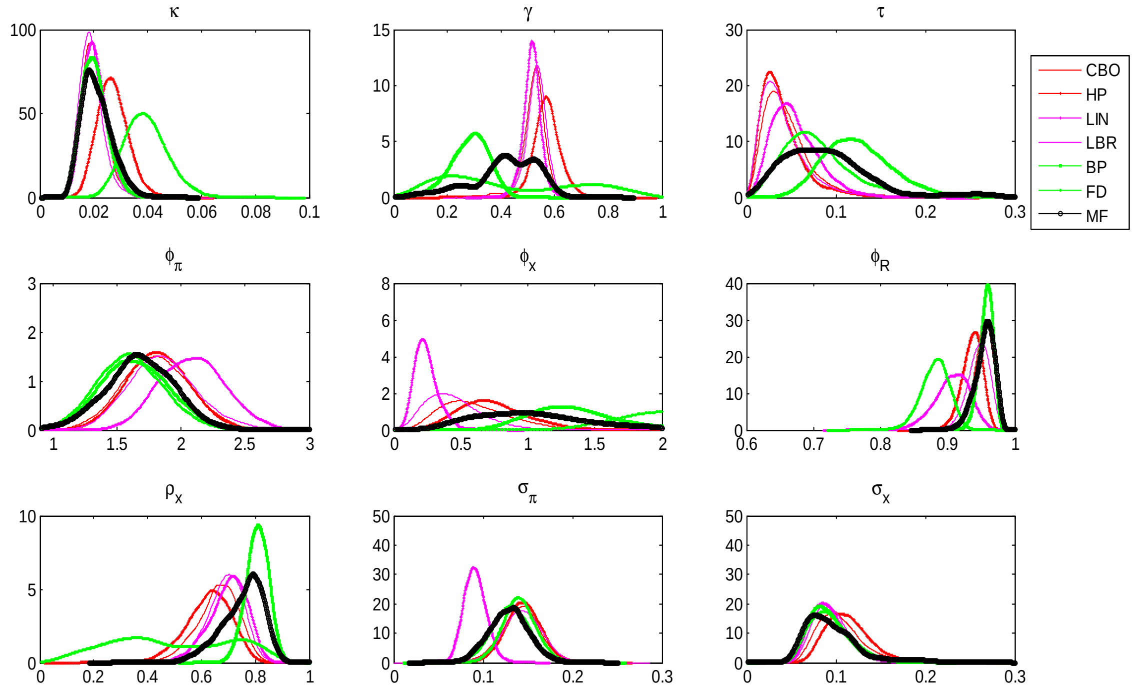

Figure I: **Structural Parameters, Posterior Densities.** Filters described in the text.

Param.	*Interpretation*	*Density*	*Priors* *Mean* *(St.dev)*	*Posteriors* *CBO*	*HP*	*LIN*	*FD*	*MF*
β	Discount factor	*Calibrated*	0.99	0.99	0.99	0.99	0.99	0.99
κ	NKPC, slope	*Gamma*	0.05 (0.01)	0.02 [0.01,0.03]	0.03 [0.01,0.04]	0.02 [0.01,0.03]	0.04 [0.03,0.05]	0.02 [0.01,0.03]
γ	ISC, forw. look. degree	*βeta*	0.5 (0.20)	0.53 [0.46,0.59]	0.57 [0.49,0.66]	0.52 [0.46,0.57]	0.36 [0.08,0.83]	0.43 [0.21,0.59]
τ	Intertemp. Elasticity of Subst.	*Gamma*	0.1 (0.05)	0.04 [0.01,0.08]	0.03 [0.01,0.07]	0.05 [0.01,0.09]	0.07 [0.02,0.14]	0.08 [0.02,0.15]
ϕ_π	TRule, react. to inflation	*Normal*	1.5 (0.30)	1.80 [1.39,2.23]	1.81 [1.41,2.22]	2.08 [1.67,2.51]	1.59 [1.16,2.02]	1.68 [1.25,2.12]
ϕ_x	TRule, react. to detr. output	*Gamma*	0.3 (0.20)	0.58 [0.21,1.06]	0.71 [0.32,1.16]	0.23 [0.10,0.38]	2.06 [1.42,2.73]	0.99 [0.35,1.70]
ϕ_R	TRule, interest rate smoothing	*βeta*	0.5 (0.20)	0.95 [0.93,0.97]	0.94 [0.91,0.96]	0.91 [0.86,0.95]	0.88 [0.84,0.92]	0.96 [0.93,0.98]
ρ_π	Infl. shock, persistence	*βeta*	0.5 (0.20)	0.20 [0.04,0.39]	0.18 [0.04,0.35]	0.63 [0.58,0.73]	0.18 [0.04,0.34]	0.24 [0.07,0.46]
ρ_x	Output shock, persistence	*βeta*	0.5 (0.20)	0.67 [0.54,0.78]	0.63 [0.49,0.76]	0.71 [0.58,0.81]	0.47 [0.16,0.84]	0.77 [0.63,0.87]
ρ_*	Infl. target, persistence	*Calibrated*	0.995	0.995	0.995	0.995	0.995	0.995
σ_π	Infl. shock, variance	*Inverse Gamma*	0.25 (2.00)	0.14 [0.11,0.18]	0.14 [0.11,0.18]	0.09 [0.07,0.11]	0.14 [0.11,0.17]	0.13 [0.09,0.16]
σ_x	Output shock, variance	*Inverse Gamma*	0.25 (2.00)	0.10 [0.06,0.14]	0.11 [0.07,0.15]	0.09 [0.06,0.12]	0.09 [0.06,0.13]	0.09 [0.05,0.13]
σ_R	MP shock, variance	*Inverse Gamma*	0.25 (2.00)	0.14 [0.12,0.15]	0.13 [0.12,0.15]	0.13 [0.11,0.15]	0.10 [0.08,0.12]	0.13 [0.12,0.15]
σ_*	Infl. target shock, variance	*Inverse Gamma*	0.25 (2.00)	0.06 [0.05,0.08]	0.06 [0.05,0.08]	0.06 [0.05,0.08]	0.06 [0.04,0.08]	0.07 [0.05,0.08]
ML_NKBC				−51.74	−37.79	−61.02	−24.90	−559.08
ML_IND				−54.09	−36.98	−52.06	−22.61	−563.43

Table I: **Structural Parameters: Prior and Posterior Densities.** 'ML': Marginal Likelihood (in logs).

	CBO	HP	LIN	LBR	BP	FD
	Standard mon. pol. shock - 16-quarter ahead					
Output	28.96	−16.51	−92.61	−13.49	78.54	−18.48
Inflation	22.60	−48.21	−92.41	−9.80	23.80	−94.59
	Standard mon. pol. shock - 40-quarter ahead					
Output	23.09	−32.21	−99.04	−13.65	46.64	−52.45
Inflation	22.37	−51.38	−94.57	−8.62	43.57	−96.27
	Trend inflation shock - 16-quarter ahead					
Output	6.97	−24.02	−97.24	−17.14	8.18	−20.93
Inflation	−1.84	−6.31	−53.37	−2.32	−9.60	−0.82
	Trend inflation shock - 40-quarter ahead					
Output	0.94	−25.36	−98.66	−16.02	−23.23	−15.40
Inflation	−1.39	−2.89	−51.55	−1.29	−13.48	1.24

Table II: **Forecast Error Variance Decomposition to Monetary Policy Shocks: Percent Deviations with respect to Multiple Filters Models.** Figures computed by relying on posterior modes.

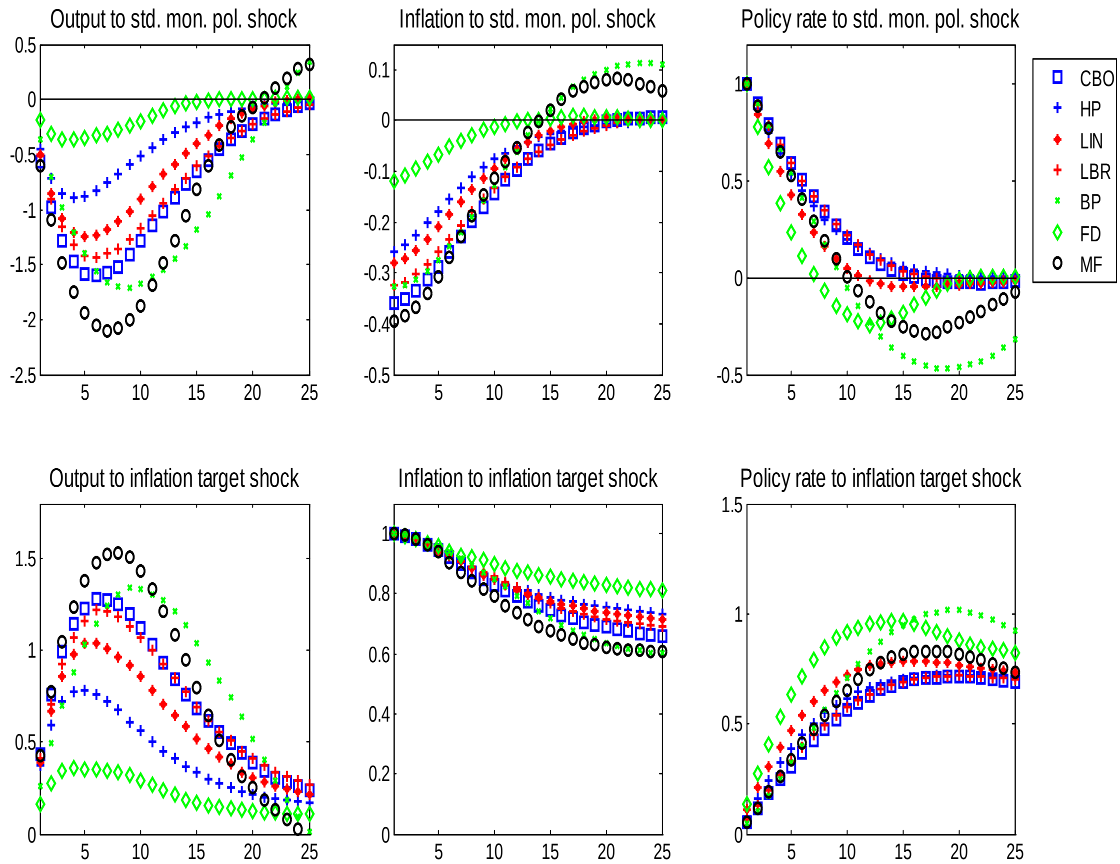

Figure II: **Impulse Response Functions to Monetary Policy Shocks - 'First Difference' Indicator Excluded from 'Multiple Filter' Model.** First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.

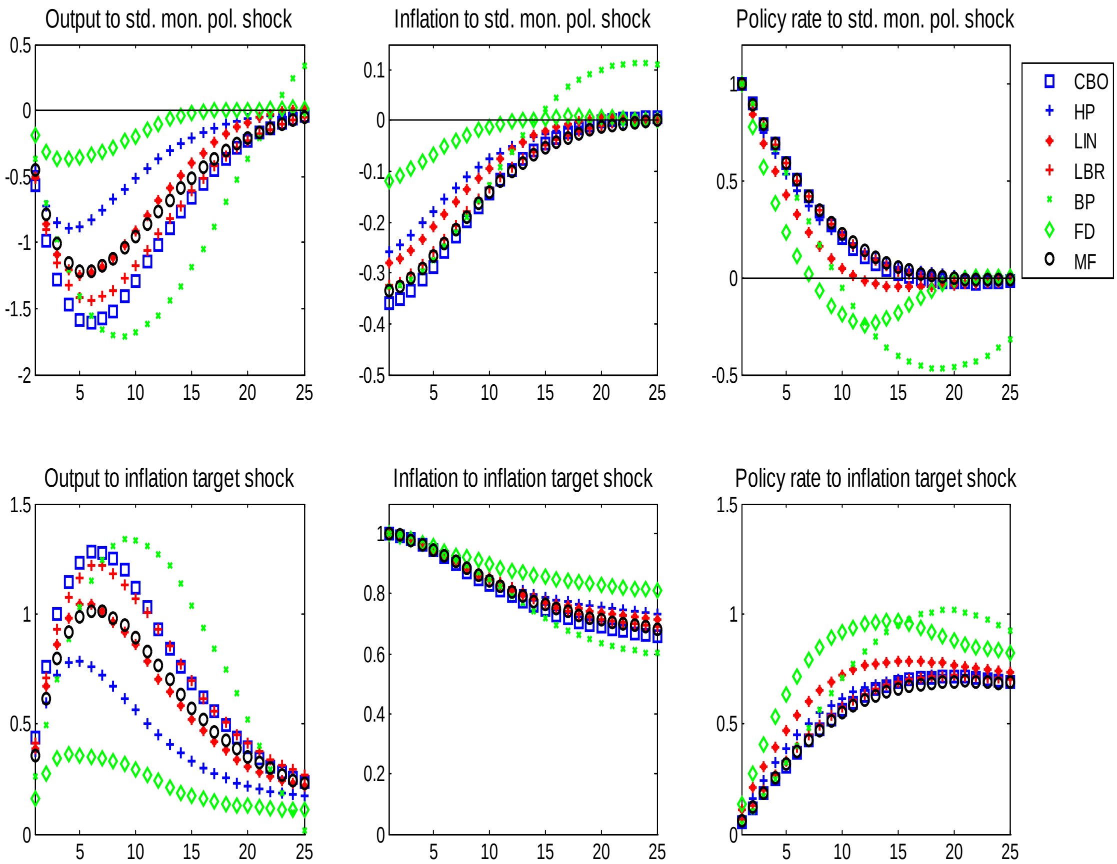

Figure III: **Impulse Response Functions to Monetary Policy Shocks - 'Band Pass' Filter Indicator Excluded from 'Multiple Filter' Model.** First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.

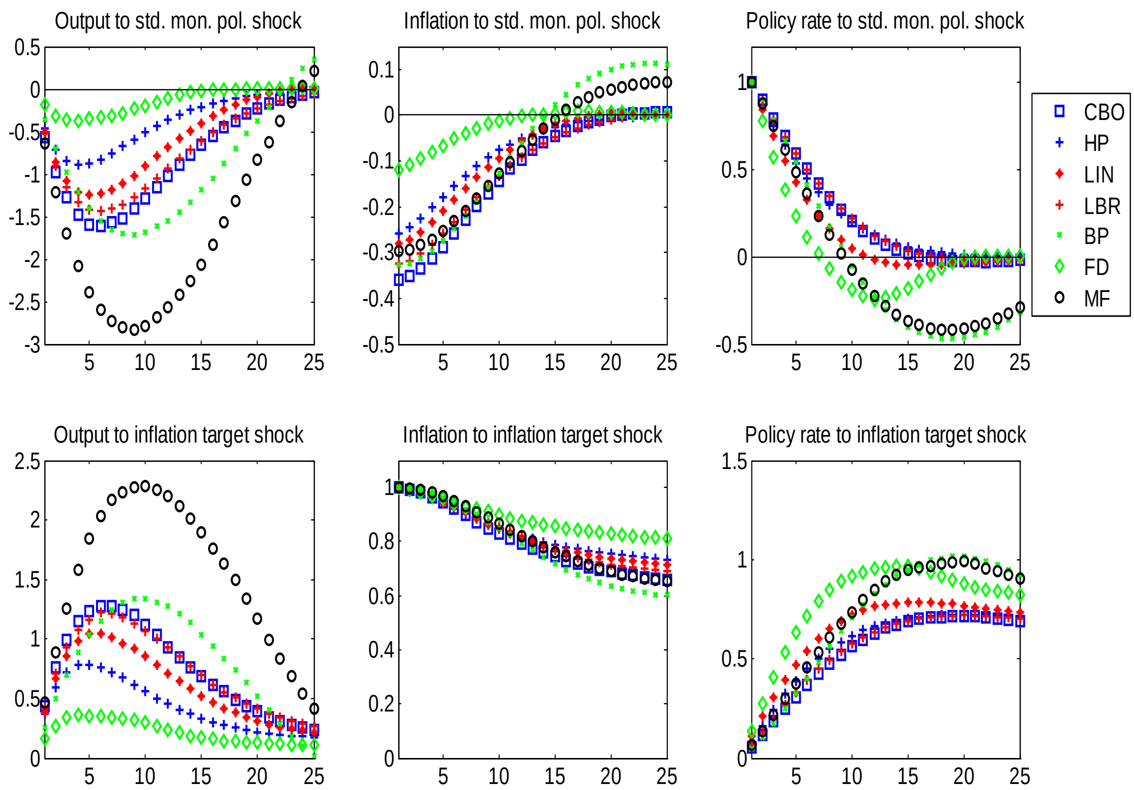

Figure IV: **Impulse Response Functions to Monetary Policy Shocks - MF with not-normalized weights**. First row: Responses to a standard monetary policy shock - normalized to induce a 1% on-impact increase in the policy rate. Last row: Responses to a trend inflation shock - normalized to induce a 1% on-impact increase in the inflation rate. Median Bayesian impulse responses based on 500 draws from the posterior densities.