7 Online Appendix

This appendix continues the Appendix section in the manuscript "On the Efficiency of Partial Information in Elections" by Jon X. Eguia and Antonio Nicolo. It uses definition, notation and results and cross-references to the Appendix section in the manuscript.

7.1 Efficiency Concerned Candidates

7.1.1 Pure strategy equilibria

Proposition 8 *For any $\beta \in \left(\frac{2}{3}, 1\right)$, the equilibrium in which candidates use the strategy pair (s_1, s_1) exists if and only if $\varepsilon \leq \frac{3-2\beta}{2}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{11}$ exists if and only if $\varepsilon \leq \frac{2-\beta}{6-4\beta}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{13}$ exists if and only if $\varepsilon \leq \frac{1}{8-6\beta}$; there are no other pure strategy equilibria.*

For any $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$, the equilibrium in which candidates use the strategy pair (s_1, s_1) exists for all $\varepsilon \geq 0$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_k$ for $k \in \{5, 6, 11\}$ exists if and only if $\varepsilon \leq \frac{1}{4-2\beta}$; the equilibrium in which candidates use the strategy $(s^A, s^B) \in S_{13}$ exists if and only $\varepsilon \leq \frac{1}{8-6\beta}$; there are no other pure strategy equilibria.

Proof. As before, to sustain equilibria, we assume off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts. We prove the high benefit case first.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence when $\varepsilon < \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. Since candidates are efficiency concerned and any deviation implies a welfare loss, they have not incentive to deviate. Suppose that $\varepsilon \geq \frac{1}{2}$. If candidate J deviates to $s^J \in \{s_2, s_3, s_4, s_8\}$ loses the election and makes a more inefficient proposal. If candidate J deviates to $s^J = s_5$ (or to $s^J = s_6$, or $s^J = s_7$) wins the election with probability ε (when full information is revealed) and therefore the deviation is profitable if and only if

$$\varepsilon \frac{1}{1+2(1-\beta)} > \frac{1}{2}, \text{ or}$$
$$\varepsilon > \frac{3-2\beta}{2}.$$

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. By Lemma 4, this cannot occur in equilibrium.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs such that $\omega_2^{i,B}(0) = 1$ for all $i \in \{a, b, c\}$, every voter votes for A. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to $s^A = s_8$, then candidate A wins the election. The deviation is profitable if only if

$$\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+(1-\beta)}$$

which holds for all $\beta \in (0, 1)$.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. If candidate A deviates to $s^A = s_6$, then candidate A wins the election. The deviation is profitable since for all β, we have that

$$\frac{1}{1+2(1-\beta)} > \frac{1}{2}\frac{1}{1+(1-\beta)}$$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Consider the deviation $s^J = s_8$. If information is not fully revealed candidate J wins the election because voter c votes for J. If information is fully revealed candidate J loses the election. Hence, candidate J prefers to deviate if and only if

$$(1-\varepsilon)\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$$
$$\varepsilon < \frac{2-\beta}{6-4\beta}$$

Consider the deviation $s^J = s_2$. If information is not fully revealed candidate J loses the election because voter b votes for J. If information is fully revealed candidate J wins the election. Hence, candidate J prefers to deviate if and only if

$$\varepsilon\frac{1}{1+(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or}$$
$$\varepsilon > \frac{2-\beta}{6-4\beta}$$

Hence there is always profitable deviation for all $\varepsilon \neq \frac{2-\beta}{6-4\beta}$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. If candidate J deviates to $s^J \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, J loses the election. If J deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, J loses the election. It follows that the best deviation for an efficiency concerned candidate is s_2 since candidate J wins the election when information is fully revealed and minimizes the number of proposed projects (if J proposes s_1 she loses the election). Candidate J prefers to deviate to $s^J = s_2$ if only if

$$\varepsilon\frac{1}{1+(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or} \tag{3}$$
$$\varepsilon > \frac{2-\beta}{6-4\beta}. \tag{4}$$

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : If candidate J deviates to any strategy $s^J \neq s_8$ and full information is not revealed, J loses the election. Hence the best deviation for an efficiency concerned candidate is s_1 because J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate to s_1 if only if

$$\varepsilon > \frac{1}{8 - 6\beta}. \tag{5}$$

Next we prove the low benefit case.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If full information is revealed there is not a different proposal that can defeat s_1. Hence there are no profitable deviation for all $\varepsilon \in [0, 1]$.

S_2 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. If candidate J deviates, she loses the election when information is not fully revealed. Hence the most profitable deviation is $s^J = s_1$ because J wins the election when information is fully revealed and the proposal is efficient. Candidate A prefers to deviate if and only if

$$\varepsilon > \frac{1}{4 - 2\beta}.$$

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given voters' beliefs, $v(s_2, s_3) = (A, B, \emptyset)$.

If J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. For the same argument as above, the best deviation for an efficiency concerned candidate J is $s^J = s_1$. Candidate J prefers to deviate if and only if

$$\varepsilon > \frac{1}{4-2\beta}.$$

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates to $s^J = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate J wins with probability $1 - \varepsilon$. Candidate J prefers to deviate if and only if

$$(1-\varepsilon)\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$$
$$\varepsilon < \frac{2+\beta}{6-2\beta}$$

Consider the deviation $s^J = s_1$. Candidate J only wins when information is fully revealed, and therefore the deviation is profitable if and only if

$$\varepsilon > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or}$$
$$\varepsilon > \frac{1}{6-4\beta}$$

Since $\frac{1}{6-4\beta} < \frac{2+\beta}{6-2\beta}$ there is always a profitable deviation.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. If J deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, J loses the election.

If J deviates to $s^J = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, J loses the election. It follows that the most profitable deviation is $s^J = s_1$ since candidate J wins the election when information is fully revealed and the proposal is efficient. Candidate J prefers to deviate to s_1 if and only if

$$\varepsilon > \frac{1}{6 - 4\beta}.$$

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. If candidate J deviates and full information is not revealed, any voter who observes the deviation votes for $-J$ and J loses the election. If candidate J deviates $s^J = s_1$, she wins the election when information is fully revealed, and this proposal is efficient. Therefore candidate J prefers to deviate to s_1 if and only if

$$\varepsilon > \frac{1}{8 - 6\beta}. \tag{6}$$

■

7.1.2 Mixed strategy Equilibria

We look at the set of mixed equilibria when pure strategy equilibria do not exist. We know that if $\beta > \frac{2}{3}$ and $\varepsilon > \frac{3}{2} - \beta$. there exists no pure strategy equilibrium, and otherwise there exists at least one. Let $s_L = (0, 1/3, 1/3, 1/3, 0, 0, 0, 0)$ and $s_H = (0, 0, 0, 0, 1/3, 1/3, 1/3, 0)$ be the special mixed strategies that consist, respectively, on proposing exactly one project and randomizing which one, and proposing exactly two projects and randomizing which two. To characterize the set of mixed equilibria when candidates are efficiency concerned, is far from being trivial. The following two propositions provide a sufficiently detailed picture of the set of mixed equilibria in the area of parameters value (ε, β) where a pure strategy equilibrium does not exist.

Proposition 9 *If $\varepsilon \in (\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta})$, candidate strategies (s_H, s_H) are supported in equilibrium. Conversely, in any symmetric equilibrium, $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$ for $J \in \{A, B\}$.*

Proof. First we prove that (s_H, s_H) can be supported in equilibrium. Given (s_H, s_H) is played, for any candidate J and voter i, if i does not observe the full proposals, beliefs are such that $\omega_1^{i,J}(1) = \omega_2^{i,J}(0) = 1$ and hence $s^i(1,0) = A$, $s^i(0,1) = B$ and $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$. Therefore, given that (s_H, s_H) is played as expected by voters, either all voters abstain if $p^A = p^B$ or one voter votes for A, one for B and one abstains if $p^A \neq p^B$; in either case the election is tied and the probability that each candidate is elected is $\frac{1}{2}$, so the expected payoff for each candidate is $\frac{1}{2}\frac{1}{3-2\beta}$.

Suppose A deviates to s_1, A loses $0-2$ if Nature does not reveal p, and A loses $1-2$ if Nature reveals p. Suppose A deviates to $s_k \in \{s_2, s_3, s_4\}$. If Nature reveals p, with probability $\frac{2}{3}$ A wins and with probability $\frac{1}{3}$ A loses; while if p is not revealed, A loses for sure. Hence, A wins with probability $\frac{2}{3}\varepsilon$, the expected utility deviating is $\frac{2}{3}\frac{\varepsilon}{2-\beta}$ and the deviation is profitable if and only if $\frac{2}{3}\frac{\varepsilon}{2-\beta} > \frac{1}{2}\frac{1}{3-2\beta}$, that is, $\varepsilon > \frac{3}{4}\frac{2-\beta}{3-2\beta}$. Suppose A deviates to $s_k \in \{s_5, s_6, s_7\}$. Then A achieves the same expected electoral outcomes and utilities as not deviating. Suppose A deviates to s_8. If Nature reveals p, A loses $1-2$. If Nature does not reveal p, then A wins $2-1$. But Nature reveals p with probability ε, hence the expected utility for A is $(1-\varepsilon)\frac{1}{4-3\beta}$ which is less than $\frac{1}{2}\frac{1}{3-2\beta}$ for any $\varepsilon > \frac{1}{2}$. Hence, there is no profitable deviation.

Second, we prove that in any symmetric equilibrium both candidates propose two projects.

Suppose (σ^J, σ^J) is part of a symmetric equilibrium. Since the equilibrium is symmetric, for any $i \in \{a, b, c\}$, if i does not observe p, then $s^i(p_i^A, p_i^B) = \varnothing$ if $p_i^A = p_i^B$ and furthermore, for $k \in \{0, 1\}$, $s^i(k, 1-k) = A$ if and only if $s^i(1-k, k) = B$. Suppose $s^i(1,0) \neq A$, so $s^i(0,1) \neq B$. Then s_8 is not a best response and it is not played in equilibrium. But if s_8 is not played, the expected payoff for i if A wins is strictly higher than if B wins given $(p_i^A, p_i^B) = (1,0)$, so by assumption $s^i(1,0) = A$, a contradiction. Thus, it must be $s^i(1,0) = A$ and $s^i(0,1) = B$. Then, given that $\varepsilon \in (\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta})$, for any strategy σ^{-J}, a best response by J must propose two projects. Thus no strategy that proposes any other number of projects can be used in a symmetric equilibrium. ■

Claim 10 *There exists an increasing function $\varepsilon(\beta)$ such that*

if $\varepsilon \in \left(\max\left\{\frac{3}{2} - \beta, \frac{3}{4}\frac{2-\beta}{3-2\beta}\right\}, \varepsilon(\beta)\right)$, there is a unique symmetric mixed strategy equilibrium, in which $\sigma^J = (\alpha_1, \alpha_1, \alpha_1, \alpha_2, \alpha_2, \alpha_2, 1-3\alpha_1-3\alpha_2)$ and the expected number of projects *is* $\frac{18\varepsilon-6}{10\varepsilon-3}$; *and*

if $\varepsilon \in (\varepsilon(\beta), 1)$, *there is a unique symmetric mixed strategy equilibrium, in which* $\sigma^J = (0, \beta_1, \beta_1, \beta_1, \beta_2, \beta_2, \beta_2, 1 - 3\beta_1 - 3\beta_2)$ *and the expected number of projects is* $\frac{12\varepsilon-3}{10\varepsilon-3} \geq \frac{9}{7}$ *and which converges to* $\frac{9}{7}$ *as* $\varepsilon \to 1$.

We find the exact functional form of $\varepsilon(\beta)$ and the weights of the mixed strategies as part of the proof.

Proof. Let (σ^A, σ^B) be a symmetric candidate strategy profile so $\sigma^A = \sigma^B$. Since the candidates' strategies are symmetric, voters' strategies must be such that $s^i(k,k) = \varnothing$ for $k \in \{0,1\}$, and $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$ or $\{s^i(1,0) = B$ and $s^i(0,1) = A\}$ or $\{s^i(1,0) = \varnothing$ and $s^i(0,1) = \varnothing\}$ for every voter i. Suppose not $\{s^i(1,0) = A$ and $s^i(0,1) = B\}$. Then given any strategy σ^{-J}, candidate J obtains a greater expected payoff playing s_1 than playing s_8, and a strictly greater payoff if $\sigma_8^{-J} > 0$. Therefore, in a symmetric mixed equilibrium, $\sigma_8^J = 0$. Then, it follows that for any voter i who observes $p_i^J = 1$ and $p_i^{-J} = 0$, the expected payoff for voter i is greater if J wins, thus by assumption, i votes J. Therefore, $s^i(1,0) = A$ and $s^i(0,1) = B$.

Next we prove that in any symmetric mixed strategy equilibrium, $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} > 0$. Suppose not. Notice that given $\varepsilon > \frac{3}{4}$ and $s^i(1,0) = A$ and $s^i(0,1) = B$, if candidate $-J$ proposes one project and candidate J proposes two, in expectation J wins the election more often, whereas if J proposes two and $-J$ proposes zero or three, J wins more often. So if candidate $-J$ never proposes one project, proposing two projects in expectation defeats any other proposal with probability more than one half. Then, any best response by candidate J to σ^{-J} with $\sigma_2^{-J} + \sigma_3^{-J} + \sigma_4^{-J} = 0$ must be such that $\sigma_5^J + \sigma_6^J + \sigma_7^J = 1$, which in turns means that any best response by J implies $\sigma_2^J + \sigma_3^J + \sigma_4^J = 1$, a contradiction.

Similarly, in any symmetric mixed strategy equilibrium, $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} > 0$. Suppose not. Any best response by J must be such that $\sigma_2^J + \sigma_3^J + \sigma_4^J = 0$, in which in turns implies that the best response by $-J$ is $\sigma_5^{-J} + \sigma_6^{-J} + \sigma_7^{-J} = 1$.

Therefore, in any symmetric mixed strategy equilibrium, both candidates propose one project, and two projects, with positive probability. But then, it must be that $\sigma_2^J = \sigma_3^J + \sigma_4^J$ and $\sigma_5^J = \sigma_6^J = \sigma_7^J$. Given that the randomization among districts (subject to choosing a number of projects) assigns equal weight to all districts, we can reduce the strategic problem to that of assigning weights to strategies s_1, s_8, s_L, s_H. The payoff matrix is as follows:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2},\frac{1}{2} & \varepsilon, 1-\varepsilon & 0, \frac{1}{3-2\beta} & \varepsilon, \frac{1-\varepsilon}{4-3\beta} \\ s_L & \frac{1-\varepsilon}{2-\beta}, \varepsilon & \frac{1}{2(2-\beta)}, \frac{1}{2(2-\beta)} & \frac{2\varepsilon}{3(2-\beta)}, \frac{3-2\varepsilon}{3(3-2\beta)} & 0, \frac{1}{4-3\beta} \\ s_H & \frac{1}{3-2\beta}, 0 & \frac{3-2\varepsilon}{3(3-2\beta)}, \frac{2\varepsilon}{3(2-\beta)} & \frac{1}{2(3-2\beta)}, \frac{1}{2(3-2\beta)} & \frac{\varepsilon}{3-2\beta}, \frac{1-\varepsilon}{4-3\beta} \\ s_8 & \frac{1-\varepsilon}{4-3\beta}, \varepsilon & \frac{1}{4-3\beta}, 0 & \frac{1-\varepsilon}{4-3\beta}, \frac{\varepsilon}{3-2\beta} & \frac{1}{2(4-3\beta)}, \frac{1}{2(4-3\beta)} \end{array}\right\}$$

Dropping the superindex (so as to use the Maple feature of Scientific Workplace to solve the equations), a symmetric equilibrium strategy with $\sigma_1^J > 0, \sigma_L^J > 0, \sigma_H^J > 0$ and $\sigma_8^J = 0$ must satisfy

i) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L = \frac{1-\varepsilon}{2-\beta}\sigma_1 + \frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}(1-\sigma_1-\sigma_L)$

ii) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L = \frac{1}{3-2\beta}\sigma_1 + \frac{3-2\varepsilon}{3(3-2\beta)}\sigma_L + \frac{1}{2(3-2\beta)}(1-\sigma_1-\sigma_L)$

iii) $\frac{1}{2}\sigma_1 + \varepsilon\sigma_L \geq \frac{1-\varepsilon}{4-3\beta}\sigma_1 + \frac{1}{4-3\beta}\sigma_L + \frac{1-\varepsilon}{4-3\beta}(1-\sigma_1-\sigma_L)$.

Solving we obtain

$$\sigma_1 = \frac{4\varepsilon + (3 - 16\varepsilon + 6\beta\varepsilon)\sigma_L}{10\varepsilon - 3\beta}$$

and

$$\sigma_L = \frac{3 - 6(1-\beta)\sigma_1}{22\varepsilon - 12\beta\varepsilon - 3} =$$

$$\sigma_L = \frac{3 - 6(1-\beta)\frac{4\varepsilon+(3-16\varepsilon+6\beta\varepsilon)\sigma_L}{10\varepsilon-3\beta}}{22\varepsilon - 12\beta\varepsilon - 3}$$

$$\sigma_L = -\frac{-9\beta + 6\varepsilon + 24\beta\varepsilon}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta\varepsilon + 120\beta\varepsilon^2 - 18}$$

which as $\beta \to 1$ converges to $\sigma_L = -\frac{-9+6\varepsilon+24\varepsilon}{60\varepsilon-100\varepsilon^2-9} = \frac{3(10\varepsilon-3)}{(10\varepsilon-3)^2} = \frac{3}{10\varepsilon-3}$ as it ought to.

Also,

$$\sigma_1 = \frac{4\varepsilon - (3 - 16\varepsilon + 6\beta\varepsilon)\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18}}{10\varepsilon - 3\beta}$$

$$\sigma_1 = -\frac{88\varepsilon^2 - 60\varepsilon + 18\beta\varepsilon - 48\beta\varepsilon^2 + 9}{9\beta + 126\varepsilon - 220\varepsilon^2 - 66\beta\varepsilon + 120\beta\varepsilon^2 - 18}$$

which as $\beta \to 1$ converges to $\sigma_1 = \frac{4\varepsilon-3}{10\varepsilon-3}$ as it ought to.

Then simplifying inequality $iii)$,

$$\frac{1}{2}\sigma_1 + \varepsilon\sigma_L \geq \frac{1-\varepsilon}{4-3\beta}\sigma_1 + \frac{1}{4-3\beta}\sigma_L + \frac{1-\varepsilon}{4-3\beta}(1-\sigma_1-\sigma_L)$$

$$\sigma_1 \geq \frac{6\varepsilon(1-\beta)\sigma_L - 2(1-\varepsilon)}{3\beta-4}$$

$$-\frac{88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} \geq \frac{-6\varepsilon(1-\beta)\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} - 2(1-\varepsilon)}{3\beta-4}$$

$$\frac{6\varepsilon(1-\beta)\frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} + 2(1-\varepsilon)}{3\beta-4} - \frac{88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} = 0$$

$$\frac{48\varepsilon-9\beta-304\varepsilon^2+440\varepsilon^3+48\beta\varepsilon+24\beta\varepsilon^2-240\beta\varepsilon^3}{(3\beta-4)(10\varepsilon-3)(3\beta+22\varepsilon-12\beta\varepsilon-6)} = 0$$

$$48\varepsilon-9\beta-304\varepsilon^2+440\varepsilon^3+48\beta\varepsilon+24\beta\varepsilon^2-240\beta\varepsilon^3 = 0$$

The solution, solved by Mathematica, is a cumbersome expression that simplifies to the desired $\varepsilon > \frac{11+\sqrt{61}}{20}$ for $\beta = 1$.

The probability of proposing two projects is

$$1 + \frac{-9\beta+6\varepsilon+24\beta\varepsilon+88\varepsilon^2-60\varepsilon+18\beta\varepsilon-48\beta\varepsilon^2+9}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} = \frac{3\left(44\varepsilon^2-24\varepsilon+8\beta\varepsilon-24\beta\varepsilon^2+3\right)}{(10\varepsilon-3)(3\beta+22\varepsilon-12\beta\varepsilon-6)}$$

and the expected number of projects is $\frac{6\left(44\varepsilon^2-24\varepsilon+8\beta\varepsilon-24\beta\varepsilon^2+3\right)}{(10\varepsilon-3)(3\beta+22\varepsilon-12\beta\varepsilon-6)} - \frac{-9\beta+6\varepsilon+24\beta\varepsilon}{9\beta+126\varepsilon-220\varepsilon^2-66\beta\varepsilon+120\beta\varepsilon^2-18} = \frac{1}{10\varepsilon-3}(12\varepsilon-3) = \frac{12\varepsilon-3}{10\varepsilon-3}$, which converges to $\frac{9}{7}$ as ε converges to 1. The initial assumption that voters vote $s^i(1,0) = A$ and $s^i(0,1) = B$ is supported because $\sigma_8 = 0$ and $\beta > 2/3$.

If instead ε is below the cutoffs, candidates mix between proposing one, two and three projects. An equilibrium with these characteristics requires:

$$\left\{\begin{array}{ccccc} & s_1 & s_L & s_H & s_8 \\ s_1 & \frac{1}{2},\frac{1}{2} & \varepsilon, 1-\varepsilon & 0, \frac{1}{3-2\beta} & \varepsilon, \frac{1-\varepsilon}{4-3\beta} \\ s_L & \frac{1-\varepsilon}{2-\beta}, \varepsilon & \frac{1}{2(2-\beta)}, \frac{1}{2(2-\beta)} & \frac{2\varepsilon}{3(2-\beta)}, \frac{3-2\varepsilon}{3(3-2\beta)} & 0, \frac{1}{4-3\beta} \\ s_H & \frac{1}{3-2\beta}, 0 & \frac{3-2\varepsilon}{3(3-2\beta)}, \frac{2\varepsilon}{3(2-\beta)} & \frac{1}{2(3-2\beta)}, \frac{1}{2(3-2\beta)} & \frac{\varepsilon}{3-2\beta}, \frac{1-\varepsilon}{4-3\beta} \\ s_8 & \frac{1-\varepsilon}{4-3\beta}, \varepsilon & \frac{1}{4-3\beta}, 0 & \frac{1-\varepsilon}{4-3\beta}, \frac{\varepsilon}{3-2\beta} & \frac{1}{2(4-3\beta)}, \frac{1}{2(4-3\beta)} \end{array}\right\}$$

$i)\ \varepsilon\sigma_L + \varepsilon(1-\sigma_L-\sigma_H) \leq \frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}\sigma_H$

$ii)\ \frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}\sigma_H = \frac{3-2\varepsilon}{3(3-2\beta)}\sigma_L + \frac{1}{2(3-2\beta)}\sigma_H + \frac{\varepsilon}{3-2\beta}(1-\sigma_L-\sigma_H)$

$iii)\frac{1}{2(2-\beta)}\sigma_L + \frac{2\varepsilon}{3(2-\beta)}\sigma_H = \frac{1}{4-3\beta}\sigma_L + \frac{1-\varepsilon}{4-3\beta}\sigma_H + \frac{1}{2(4-3\beta)}(1-\sigma_L-\sigma_H).$

Simplifying the first inequality expressions, we get $\varepsilon \leq \frac{1}{2(2-\beta)}\sigma_L + \frac{(8-3\beta)}{3(2-\beta)}\varepsilon\sigma_H$.

From ii)

$$\sigma_L = \frac{12\varepsilon - 6\beta\varepsilon + (14\beta\varepsilon + 6 - 3\beta - 24\varepsilon)\sigma_H}{20\varepsilon - 10\beta\varepsilon - 3}$$

From iii)

$$\sigma_H = -\frac{(3\beta + 6\sigma_L - 6\beta\sigma_L - 6)}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6}$$

So,

$$\begin{aligned}
\sigma_L &= \frac{12\varepsilon - 6\beta\varepsilon - (14\beta\varepsilon + 6 - 3\beta - 24\varepsilon)\frac{(3\beta+6\sigma_L-6\beta\sigma_L-6)}{3\beta+28\varepsilon-18\beta\varepsilon-6}}{20\varepsilon - 10\beta\varepsilon - 3} \\
\sigma_L &= \frac{9\beta + 108\varepsilon - 168\varepsilon^2 - 60\beta\varepsilon + 108\beta\varepsilon^2 - 18}{18\beta + 174\varepsilon - 280\varepsilon^2 - 114\beta\varepsilon + 180\beta\varepsilon^2 - 27}
\end{aligned}$$

which simplifies to

$$\sigma_L = \frac{9 + 108\varepsilon - 168\varepsilon^2 - 60\varepsilon + 108\varepsilon^2 - 18}{18 + 174\varepsilon - 280\varepsilon^2 - 114\varepsilon + 180\varepsilon^2 - 27} = \frac{6\varepsilon - 3}{10\varepsilon - 3}$$

when $\beta = 1$ as desired. Also,

$$\begin{aligned}
\sigma_H &= -\frac{3\beta - 6 + 6(1-\beta)\sigma_L}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6} = -\frac{3\beta - 6 + 6(1-\beta)\frac{9\beta+108\varepsilon-168\varepsilon^2-60\beta\varepsilon+108\beta\varepsilon^2-18}{18\beta+174\varepsilon-280\varepsilon^2-114\beta\varepsilon+180\beta\varepsilon^2-27}}{3\beta + 28\varepsilon - 18\beta\varepsilon - 6} \\
\sigma_H &= \frac{1}{10\varepsilon - 3}\frac{24\varepsilon + 6\beta\varepsilon - 9}{6\beta + 28\varepsilon - 18\beta\varepsilon - 9}
\end{aligned}$$

which simplifies to

$$\frac{1}{10\varepsilon - 3}\frac{24\varepsilon + 6\varepsilon - 9}{6 + 28\varepsilon - 18\varepsilon - 9} = \frac{3}{10\varepsilon - 3}$$

and, the expected number of projects in this equilibrium is:

$$\begin{aligned}
& 2\frac{1}{10\varepsilon-3}\frac{24\varepsilon+6\beta\varepsilon-9}{6\beta+28\varepsilon-18\beta\varepsilon-9}+\frac{9\beta+108\varepsilon-168\varepsilon^2-60\beta\varepsilon+108\beta\varepsilon^2-18}{18\beta+174\varepsilon-280\varepsilon^2-114\beta\varepsilon+180\beta\varepsilon^2-27} \\
& +3(1-\frac{24\varepsilon+6\beta\varepsilon-9-(9\beta+108\varepsilon-168\varepsilon^2-60\beta\varepsilon+108\beta\varepsilon^2-18)}{(10\varepsilon-3)(6\beta+28\varepsilon-18\beta\varepsilon-9)}) \\
= \; & \frac{18\varepsilon-6}{10\varepsilon-3}.
\end{aligned}$$

■

7.2 One Office Motivated and one Efficiency Concerned candidate

We characterize the set of pure equilibria when one candidate is office motivated and the other one is efficiency concerned.

Proposition 11 *If projects are not very inefficient* $\beta \in (\frac{2}{3}, 1)$, *there exist a cutoff function* $\varepsilon_1(\beta)$ *such* $0 < \varepsilon_1(\beta) < \frac{1}{2}$ *and:*

If $\varepsilon \in [0, \varepsilon_1(\beta)]$, *there exist multiple pure equilibria;*

If $\varepsilon \in (\varepsilon_1(\beta), \frac{1}{2})$, *there is a unique pure strategy equilibrium in which both candidates propose the efficient policy; and*

If $\varepsilon > \frac{1}{2}$ *there is no pure strategy equilibrium.*

If projects are very inefficient $\beta \in (\frac{1}{3}, \frac{2}{3})$, *there exists a cutoff function* $\varepsilon_1(\beta)$ *such* $0 < \varepsilon_1(\beta) < \frac{1}{2}$ *and:*

If $\varepsilon \in [0, \varepsilon_1(\beta)]$, *there exist multiple pure equilibria;*

If $\varepsilon \in (\varepsilon_1(\beta), 1]$ *in the unique pure equilibrium both candidates propose the efficient policy.*

Proof. Without loss of generality, let assume that candidate A is office motivated. We prove the high benefit case first.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied and if candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. If $\varepsilon \leq \frac{1}{2}$, no hypothetical gain when full information is revealed can compensate for this loss. If a candidate is efficiency concerned, deviations give her a lower payoff so there is not incentive to deviate. Suppose now that $\varepsilon > \frac{1}{2}$. By assumption candidate A is office motivated. If A deviates to $s^A = s_5$ she wins the election with probability ε (when full information is revealed) and therefore such deviation is profitable for the office motivated candidate.

Candidate A is purely office motivated and candidate B is efficiency concerned. The set S_2 contains profiles which are not strategically equivalent because candidate B is not indifferent, for instance, between winning with proposals s_1 or s_2. We must partition the set S_2 into the following subsets of profiles which are equivalent for both players, $S_2' = \{(s_1, s_2), (s_1, s_3), (s_1, s_4), \}$ and $S_2'' = \{(s_2, s_1), (s_3, s_1), (s_4, s_1)\}$

S_2' : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

$S_2'' =$ Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Ruled out by Lemma 4.

Similary as before we partition S_3 in two subsets $S_3' = \{(s_1, s_5), (s_1, s_6), (s_1, s_7)\}$ and $S_3'' = \{(s_5, s_1), (s_6, s_1), (s_7, s_1)\}$.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Ruled out by Lemma 4.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (A, A, B)$. Ruled out by Lemma 4.

(s_1, s_8) : Every voter votes for A. Ruled out by Lemma 4.

(s_8, s_1) : Every voter votes for B. Ruled out by Lemma 4.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to $s^J = s_8$ wins the election both in case the information is revealed and in case it is not.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ both in case full information is revealed and in case it not revealed, A

wins the election. Hence the deviation is profitable for all $\varepsilon \geq 0$.

Let $S'_7 = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S''_7 = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S'_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S''_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. Ruled out by Lemma 4.

Let $S'_8 = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S''_8 = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S'_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S''_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. Ruled out by Lemma 4.

Let $S'_9 = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S''_9 = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S'_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. Ruled out by Lemma 4.

S''_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, B, A)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. Suppose first that $\varepsilon < \frac{1}{2}$. If candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose that $\varepsilon > \frac{1}{2}$. If A deviates to $s^A = s_2$ then A wins the election when information is fully revealed. Hence the deviation is profitable.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given $(s^A, s^B) = (s_5, s_6)$, beliefs such that $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$ support an equilibrium in which $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. Consider first the office motivated candidate A. It suffices to check that A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider now candidate B who, by assumption, is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. Playing any other deviation candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_2$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed.(playing $s^B = s_4$ gives the same payoff as s_2 when information is fully revealed, but if B plays s_4 she looses 3-0 when information is not fully revealed while if B plays s_1 she looses $2-1$) Candidate B prefers to deviate to s_2 if and only if

$$\varepsilon \frac{1}{1+(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}, \text{ or} \tag{7}$$

$$\varepsilon > \frac{2-\beta}{6-4\beta} \tag{8}$$

Hence there is a profitable deviation for candidate B if the previous condition holds. Suppose now that $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_2$, then A wins the election when full information is revealed. Hence the deviation is profitable.

Let $S'_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S''_{12} = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. Ruled out by Lemma 4.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. Ruled out by Lemma 4.

S_{13} : Let voter strategy $s^i = (\emptyset, B, A, \emptyset)$ for each voter i and beliefs such that $\omega_2^{i,J}(0) = \omega_2^{i,J}(1) = 1$ for any voter i. Suppose first that $\varepsilon \leq \frac{1}{2}$. In equilibrium the election is tied; consider the office motivated candidate A. If candidate A deviates to any strategy $s^A \neq s_8$ and full information is not revealed, A loses the election. Consider the efficiency concerned candidate B. If information is not fully revealed, candidate B looses the election if she deviates. If information is fully revealed, the unique profitable deviation is $s^B = s_1$. Candidate B prefers to deviate if and only if

$$\varepsilon > \frac{1}{8-6\beta} \tag{9}$$

Hence there is a profitable deviation for B if the previous condition holds. Suppose that $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_0$, then A wins the election when full information is revealed. Hence the deviation is profitable.

Next we prove the low benefit case. To sustain equilibria, assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observe a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ for each voter i and beliefs such that $\omega_0^{i,J}(0) = 1$ and $\omega_2^{i,J}(1) = 1$ for any voter i and any candidate J make the election tied. There is no a different proposal that can defeat s_1 both in case the information is revealed and in case it is not. The efficiency concerned candidate has lower incentive to deviate because s_1 is the efficient proposal. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium. Hence no candidate has profitable deviations for all $\varepsilon \in [0, 1]$.

S_2' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Ruled out by Lemma 4.

$S_2'' =$ Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Ruled out by Lemma 4.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. Ruled out by Lemma 4.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (B, B, B)$. Ruled out by Lemma 4.

(s_1, s_8) : Every voter votes for A. Ruled out by Lemma 4.

(s_8, s_1) : Every voter votes for B. Ruled out by Lemma 4

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given (s_2, s_2), every voter abstains. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Consider candidate B. If candidate B deviates, B looses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate B is $s^B = s_1$, since B wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\varepsilon > \frac{1}{4-2\beta}.$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Suppose $\varepsilon \leq \frac{1}{2}$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Any deviation makes the candidate looses the election when information is not fully revealed. When information is revealed the most profitable deviation for candidate B is $s^B = s_1$, since she wins the election and the proposal is efficient. The deviation $s^B = s_1$ is profitable if and only if

$$\varepsilon > \frac{1}{4-2\beta}.$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Ruled out by Lemma 4.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. Ruled out by Lemma 4.

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. Ruled out by Lemma 4.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. Ruled out by Lemma 4.

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A. Ruled out by Lemma 4.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, B, B)$. Ruled out by Lemma 4.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given (s_5, s_5), all voters abstain. If the office motivated candidate A deviates to $s^A = s_8$ and full information is not revealed, only voter c observes the deviation and $v(s_8, s_5) = (\emptyset, \emptyset, A)$. Hence by deviating candidate A wins with probability at least $1 - \varepsilon > \frac{1}{2}$. Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Suppose first that $\varepsilon \leq \frac{1}{2}$. Given (s_5, s_6), $v(s_5, s_6) = (\emptyset, A, B)$. The office motivated candidate A has no incentives to deviate. If A deviates to $s^A \in \{s_1, s_2, s_3, s_4, s_6, s_7\}$ and full information is not revealed, A loses the election. If A deviates to $s^A = s_8$ and full information is not revealed, the election is tied, but if full information is revealed, A loses the election. In any case, after a deviation A wins the election with probability less than $\frac{1}{2}$. Consider candidate B who is efficiency concerned. Deviating to $s^B = s^8$ is clearly unprofitable. By deviating candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed. Candidate B prefers to deviate to s_1 if and only if

$$\varepsilon > \frac{1}{6 - 4\beta}$$

Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_6) = (A, A, B)$. Ruled out by Lemma 4.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. Ruled out by Lemma 4.

S_{13} : All voters abstain and the election is tied. Suppose $\varepsilon \leq \frac{1}{2}$. If the office motivated candidate A deviates and full information is not revealed, any voter who observes the deviation votes for B and A loses the election. Consider candidate B who is efficiency concerned. By deviating candidate B loses the election when information is not fully revealed. So the best deviation is $s^B = s_1$ because it minimizes the inefficiency and candidate B wins the election when information is fully revealed. Candidate B prefers to deviate to $s^B = s_1$ if and only if

$$\varepsilon > \frac{1}{8-6\beta} \tag{10}$$

Hence there is a profitable deviation for candidate B if the previous condition holds. Suppose now $\varepsilon > \frac{1}{2}$. If candidate A deviates to $s^A = s_1$, A wins the election when full information is revealed. Hence the deviation is profitable. ■

The proofs of Proposition 3 and of Theorem 2 follow by the previous results. To conclude the proof of Proposition 1 we show in the following lemma that there is no equilibrium in mixed strategies when $\beta \in (\frac{1}{3}, \frac{2}{3})$ and $\varepsilon > \frac{1}{2}$.

Lemma 12 *For any $(\alpha_i, \alpha_j) \in \{0,1\}^2$ and any $\beta \in (\frac{1}{3}, \frac{2}{3})$, if $\varepsilon > \frac{1}{2}$ there is no equilibrium in which a candidate proposes to provide a public good with positive probability.*

Proof. Suppose candidate J plays with positive probability any strategy different than s_1. If candidate $-J$ plays s_1, then candidate J looses the election when full information is revealed. If candidate $-J$ is playing any strategy $s_k^{-J} \neq s_1^{-J}$ then candidate J wins with probability one by playing s_1 when full information is revealed. Therefore if candidate J replace in the mixed strategy any $s_k^J \neq s_1^J$ with strategy s^J increases her probability of winning the election. ■

7.3 Equilibria with $\varepsilon = 0$

In this subsection we characterize the set of Bayesian equilibria when $\varepsilon = 0$ and voters do not play weakly dominated strategies.

Lemma 13 *Assume $\varepsilon = 0$. Every strategy is undominated.*

Proof. No candidate strategy is weakly dominated, because the payoffs to candidates depend on the strategies of the voters.

For the voters, consider the generic information set $(p_i^A, p_i^B) = (x, y)$ with $x, y \in \{0,1\}$. If $s^b(x,y) = s^c(x,y) = \emptyset$, s^J consists of proposing $(x,0,0)$, and s^{-J} consists of proposing $(y,1,1)$, then a is strictly better off voting for J, while if s^J consists of $(x,1,1)$ and s^{-J} consists of $(y,0,0)$, then a is strictly better off voting for $-J$. Thus, any strategy is undominated. ■

7.3.1 Office Motivated Candidates

Proposition 14 *Assume $\varepsilon = 0$, candidates are office motivated and $\beta \in \left(\frac{2}{3}, 1\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \in S_k$ for some $k \in \{1, 2, 3, 4, 8, 11, 13\}$.*

Proof. For each strategy pair class, we find whether an element of the class can be sustained in equilibrium. To sustain equilibria, we assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Given the beliefs, neither candidate can improve her electoral outcome by deviating.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs, in equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If A deviates to $s^A = s_6$,only voter c observes the deviation so $v(s_6, s_2) = (\emptyset, \emptyset, A)$ and candidate A wins the election.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$, only voter c observes the deviation, so $v(s_6, s_2) = (A, B, A)$ and candidate A wins the election.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. If B deviates to $s^B = s_7$, only voter c observes the deviation, so $v(s_2, s_7) = (A, B, B)$ and candidate B finds the deviation profitable.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. In equilibrium in which $v(s_2, s_7) = (A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate A deviates to s_8, voters b and c either vote for candidate A or abstains, depending upon their beliefs, and therefore candidate A wins the election (voter $a's$ beliefs do not change).

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates and proposes s_8, beliefs of voters a and b are unaffected, while voter c votes for candidate A for all possible beliefs over candidate $A's$ strategy. Therefore candidate A wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. In equilibrium $v(s_5, s_6) = (\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If A deviates to s_8, voter c either votes for A as well, or abstains, hence A is better off deviating because A increases the vote margin.

S_{13} : In equilibrium the election is tied, and if candidate J deviates to any strategy $s^J \neq s_8$, J loses the election. ■

Proposition 15 *Assume $\varepsilon = 0$, candidates are office motivated and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12}$.*

Proof. First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates and proposes s_8 voters a and b beliefs are unaffected, while voter c votes for candidate J. Therefore candidate J wins the election.

Similarly, $(s^A, s^B) \in S_{12}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), voters a and b vote A, while voter c votes B. Suppose A

deviates to s_8. Voters a and b do not observe the deviation, and continue to vote A, while voter c now abstains. Hence now A wins the election by a greater margin.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal s_i^J, then $\omega_2^{i,J}(1-s_i^J)=1$ for both $J\in\{A,B\}$ and for $i\in\{a,b,c\}$. These are most pessimistic beliefs that a voter can have regarding candidates' strategy when she observes a deviation.

S_1 : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

S_2 : Assume without loss of generality that $(s^A,s^B)=(s_1,s_2)$. In equilibrium, $v(s_1,s_2)=(A,B,B)$, and any voter who observes a deviation votes against the candidate who deviates.

S_3 : Assume w.l.o.g. that $(s^A,s^B)=(s_1,s_5)$. In equilibrium, every voter votes for A and continues to vote for A after any deviation by B.

S_4 : Assume w.l.o.g. that $(s^A,s^B)=(s_1,s_8)$. In equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A,s^B)=(s_2,s_2)$. Given (s_2,s_2), every voter abstains, and votes against any candidate who deviates.

S_6 : Assume w.l.o.g. that $(s^A,s^B)=(s_2,s_3)$. Given (s_2,s_3), $v(s_2,s_3)=(A,B,\emptyset)$. Every voter votes against any deviating candidate.

S_7 : Assume w.l.o.g. that $(s^A,s^B)=(s_2,s_5)$. Given (s_2,s_5), $v(s_2,s_5)=(A,B,A)$. Voters a and c do not vote for B after any deviation by B, and voter b does not vote for A after any deviation.

S_8 : Assume w.l.o.g. that $(s^A,s^B)=(s_2,s_7)$. Given (s_2,s_7), $v(s_2,s_7)=(A,B,B)$, and given any deviation by J, no voter changes her vote from voting for $-J$ to abstention or voting for J.

S_9 : Assume w.l.o.g. that $(s^A,s^B)=(s_2,s_8)$. All three voters vote for A and continue to do so after any deviation by B.

S_{10} : Not an equilibrium as shown above.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_g) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Not an equilibrium as shown above.

S_{13} : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

7.3.2 Efficiency Concerned Candidates

Proposition 16 *Assume* $\varepsilon = 0$,*candidates are efficiency concerned and* $\beta \in \left(\frac{2}{3}, 1\right)$. *An equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 2, 3, 4, 8, 11, 12, 13\}$.

Proof. To sustain equilibria, we assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, $v(s_1, s_5) = (B, B, A)$. Given the beliefs, neither candidate can improve her electoral outcome by deviating.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. Given beliefs, in equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If A deviates to $s^A = s_6$,only voter c observes the deviation so $v(s_6, s_2) = (\emptyset, \emptyset, A)$ and candidate

A wins the election. Candidate A prefers to deviate if and only if $\frac{1}{1+(1-\beta)2} > \frac{1}{2}\frac{1}{1+(1-\beta)}$which holds for all $\beta < 1$.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$, only voter c observes the deviation, so $v(s_6, s_2) = (A, B, A)$ and candidate A wins the election. As proved above, all candidate candidate A finds the deviation profitable.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. If B deviates to $s^B = s_7$, only voter c observes the deviation, so $v(s_2, s_7) = (A, B, B)$ and candidate B finds the deviation profitable.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. In equilibrium in which $v(s_2, s_7) = (A, B, B)$. It is easy to check that no candidate can improve her electoral outcome with any deviation.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate A deviates to s_8, voters b and c abstain and therefore candidate A wins the election (voter $a's$ beliefs do not change), so candidate $A's$ finds profitable to deviate.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates and proposes s_8, beliefs of voters a and b are unaffected, while voter c votes for candidate A for all possible beliefs over candidate $A's$ strategy. Therefore candidate A wins the election. Candidate A finds profitable to deviate if and only if $\frac{1}{1+(1-\beta)3} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$ which holds for all $\beta < 1$.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. In equilibrium $v(s_5, s_6) = (\emptyset, A, B)$ and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. By deviating candidate B cannot increase the number of votes he gets, due to voters' beliefs. For candidate A, the unique strategy that increases A's vote margin is s_8 because voter c would abstain, but an office motivated candidate does not find this deviation profitable.

S_{13} : In equilibrium the election is tied, and if candidate J deviates to any strategy $s^J \neq s_8$, J loses the election. ■

Proposition 17 *Assume $\varepsilon = 0$, candidates are efficiency concerned and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin$*

S_{10}.

Proof. First note that $(s^A, s^B) \in S_{10}$ cannot be supported in equilibrium. Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate J deviates and proposes s_8 voters a and b beliefs are unaffected, while voter c votes for candidate J for all possible beliefs over candidate J strategy. Therefore candidate J wins the election. This deviation is profitable if and only if $\frac{1}{1+3(1-\beta)} > \frac{1}{2}\frac{1}{1+2(1-\beta)}$ which holds for all $\beta < 1$.

All the other strategy profiles are sustained in equilibria by the following beliefs. Beliefs' over equilibrium strategies are correct. Out-of-equilibrium beliefs are such that given the equilibrium proposal s_i^J, then $\omega_2^{i,J}(1 - s_i^J) = 1$ for both $J \in \{A, B\}$ and for $i \in \{a, b, c\}$.

S_1 : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates.

S_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. In equilibrium, $v(s_1, s_2) = (A, B, B)$, and any voter who observes a deviation votes against the candidate who deviates.

S_3 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. In equilibrium, every voter votes for A and continues to vote for A after any deviation by B.

S_4 : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_8)$. In equilibrium every voter votes for A and continues to vote for A after any deviation by B.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter abstains, and votes against any candidate who deviates.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. Every voter votes against any deviating candidate.

S_7 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. Voters a and c do not vote for B after any deviation by B, and voter b does not vote for A after any deviation.

S_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$, and given any deviation by J, no voter changes her vote from voting for $-J$ to abstention or voting for J.

S_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. All three voters vote for A and continue to do so after any deviation by B.

S_{10} : Not an equilibrium as shown above.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Given (s_5, s_6), $v(s_5, s_g) = (\emptyset, A, B)$, and it is again easy to check that no candidate can gain any vote by deviating.

S_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. By deviating candidate B cannot increase the number of votes he gets, due to voters' beliefs. For candidate A, the unique strategy that increases A's vote margin is s_8 because voter c would abstain, but an office motivated candidate does not find this deviation profitable.

S_{13} : All voters abstain and the election is tied. Any voter who observes a deviation votes against the candidate who deviates. ■

7.3.3 One efficiency concerned, one office motivated candidate

Proposition 18 *Assume* $\varepsilon = 0$, *one candidate is efficiency concerned and the other is office motivated, and* $\beta \in \left(\frac{2}{3}, 1\right)$. *An equilibrium in which candidates use the strategy pair* (s^A, s^B) *exists if and only if* $(s^A, s^B) \in S_k$ *for some* $k \in \{1, 2, 3, 4, 8, 11, 13\}$ *and if* $(s^A, s^B) \in S''_{12}$ *where* $S''_{12} = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$

Proof. Supose without loss of generality that candidate A is office motivated. We assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : Voter strategy $s^i = (\emptyset, A, B, \emptyset)$ and the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S'_2 : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

S''_2 = Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Given voters' beliefs, candidate A cannot win the election by deviating and candidate B cannot increase her vote margin by deviating.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (B, B, A)$. Candidate A cannot win the election deviating to any strategy $s_A \neq s_1$. Candidate B is efficiency concerned so he might be interested in deviating to reduce the number of projects he proposes. By deviating to s_2, the election is tied: candidate B prefers to deviate if and only if $\frac{1}{2}\frac{1}{1+(1-\beta)} > \frac{1}{1+2(1-\beta)}$, but this condition never occurs. By deviating to s_1 candidate B looses the election so this deviation is never profitable.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (A, A, B)$. No candidate has a profitable deviation given voters' beliefs.

(s_1, s_8) : Every voter votes for A. No candidate has a profitable deviation given voters' beliefs.

(s_8, s_1) : Every voter votes for B. No candidate has a profitable deviation given voters' beliefs.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. If candidate A deviates to s_8 wins the election.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. If A deviates to $s^A = s_6$ both in case full information is revealed and in case it not revealed, A wins the election.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. No candidate has a profitable deviation given voters' beliefs.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S_8'' : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_9' = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S_9'' = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S_9' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S_9'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates to s_8 wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Then $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. No candidate has a profitable deviation given voters' beliefs.

Let $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S_{12}'' = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S_{12}' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If candidate A deviates to s_8, then voter c abstains and therefore candidate A wins with greater margin.

S_{12}'' : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. No candidate has a profitable deviation given voters' beliefs, because candidate B is efficiency concerned and does not find deviation s_8 profitable even if he would win with a greater vote margin.

S_{13} : In equilibrium the election is tied; if a candidate deviates to any strategy different than s_8 looses the election. ■

Proposition 19 *Assume $\varepsilon = 0$, one candidates efficiency concerned and the other is office motivated, and $\beta \in \left(\frac{1}{3}, \frac{2}{3}\right)$. An equilibrium in which candidates use the strategy pair (s^A, s^B) exists if and only if $(s^A, s^B) \notin S_{10} \cup S_{12'}$ where $S_{12}' = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$.*

Proof. Supose without loss of generality that candidate A is office motivated. We assume that off-equilibrium path beliefs such that given the equilibrium proposal s_i^J, $\omega_2^{i,J}(1 - s_i^J) = 1$ for each $i \in \{a, b, c\}$ and $J \in \{A, B\}$. That is, a voter who observes a deviation believes that the deviating candidate proposes to carry out the projects in the other two districts.

S_1 : In equilibrium the election is tied. If candidate J deviates to any $s^J \neq s_1$, then J loses the election. It is also straightforward to check that the voting strategy is a best response given the strategy of the candidates and the beliefs of the voters, and that the beliefs are correct along the equilibrium path, so these strategies and beliefs are an equilibrium.

S_2' : Assume without loss of generality that $(s^A, s^B) = (s_1, s_2)$. Given (s_1, s_2), $v(s_1, s_2) = (B, A, A)$. Given voters' beliefs, candidate B cannot win the election by deviating and candidate A cannot increase her vote margin by deviating.

$S_2'' =$ Assume without loss of generality that $(s^A, s^B) = (s_2, s_1)$. Given (s_2, s_1), $v(s_2, s_1) = (A, B, B)$. Given voters' beliefs, candidate A cannot win the election by deviating and candidate B cannot increase her vote margin by deviating.

S_3' : Assume w.l.o.g. that $(s^A, s^B) = (s_1, s_5)$. Given (s_1, s_5), $v(s_1, s_5) = (A, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_3'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_1)$. Given (s_5, s_1), $v(s_5, s_1) = (B, B, B)$. No candidate has a profitable deviation given voters' beliefs.

(s_1, s_8) : Every voter votes for A. No candidate has a profitable deviation given voters' beliefs.

(s_8, s_1) : Every voter votes for B. No candidate has a profitable deviation given voters' beliefs.

S_5 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_2)$. Given (s_2, s_2), every voter i abstains. No candidate has a profitable deviation given voters' beliefs.

S_6 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_3)$. Given (s_2, s_3), $v(s_2, s_3) = (A, B, \emptyset)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_7' = \{(s_2, s_5), (s_2, s_6), (s_3, s_5), (s_3, s_7), (s_4, s_6), (s_4, s_7), \}$

and $S_7'' = \{(s_5, s_2), (s_6, s_2), (s_5, s_3), (s_7, s_3), (s_6, s_4), (s_7, s_4)\}$.

S_7' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_5)$. Given (s_2, s_5), $v(s_2, s_5) = (A, B, A)$. No candidate has a profitable deviation given voters' beliefs.

S_7'' : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_2)$. Given (s_5, s_2), $v(s_5, s_2) = (B, A, B)$. No candidate has a profitable deviation given voters' beliefs.

Let $S_8' = \{(s_2, s_7), (s_3, s_6), (s_4, s_5), \}$ and $S_8'' = \{(s_7, s_2), (s_6, s_3), (s_5, s_4)\}$

S_8' : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_7)$. Given (s_2, s_7), $v(s_2, s_7) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S''_8 : Assume w.l.o.g. that $(s^A, s^B) = (s_7, s_2)$. Given (s_7, s_2), $v(s_7, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

Let $S'_9 = \{(s_2, s_8), (s_3, s_8), (s_4, s_8)\}$ and $S''_9 = \{(s_8, s_2), (s_8, s_3), (s_8, s_4)\}$;

S'_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_2, s_8)$. Given (s_2, s_8), $v(s_2, s_8) = (A, B, B)$. If candidate B deviates to any strategy with less projects looses the election.

S''_9 : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_2)$. Given (s_8, s_2), $v(s_8, s_2) = (B, A, A)$. No candidate has a profitable deviation given voters' beliefs.

S_{10} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_5)$. Given (s_5, s_5), all voters abstain. If candidate A deviates to s_8 wins the election.

S_{11} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_6)$. Then $v(s_5, s_6) = (\emptyset, A, B)$ and each candidate wins with equal probability. No candidate has a profitable deviation given voters' beliefs.

Let $S'_{12} = \{(s_5, s_8), (s_6, s_8), (s_7, s_8)\}$ and $S''_{12} = \{(s_8, s_5), (s_8, s_6), (s_8, s_7)\}$;

S'_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_5, s_8)$. Given (s_5, s_8), $v(s_5, s_8) = (A, A, B)$. If candidate A deviates to s_8, then voter c abstains and therefore candidate A wins with greater margin.

S''_{12} : Assume w.l.o.g. that $(s^A, s^B) = (s_8, s_5)$. Given (s_8, s_5), $v(s_8, s_5) = (A, B, B)$. No candidate has a profitable deviation given voters' beliefs.

S_{13} : In equilibrium the election is tied; if a candidate deviates to any strategy different than s_8 looses the election. ■