On the Dynamics of International Inflation*

Giovanni Caggiano
University of Padua

Efrem Castelnuovo
University of Padua

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Abstract

We investigate the dynamic properties of inflation in 20 OECD countries with a novel approach based on the AutoCorrelation Function. We find evidence in favor of long memory and nonlinearity. Linear autoregressive models are shown to be misspecified.

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*We thank Eric Maskin (Editor) and an anonymous referee for useful comments. All remaining errors are ours. *Corresponding author*: Efrem Castelnuovo, Department of Economics, University of Padua, Via del Santo 33, I-35123 Padova (PD). Phone: +39 049 8274257; Fax: +39 049 8274211. E-mail: efrem.castelnuovo@unipd.it.

1 Introduction

This paper investigates the dynamic properties of inflation using an approach based on the AutoCorrelation Function (ACF). The approach, which is novel in the empirical literature on inflation, allows to accounting explicitly for the potential *long memory and nonlinearities* of the Data Generating Process (DGP) of the series and of its autocorrelation structure, due to sectorial heterogeneity.

The role played by heterogeneity and aggregation on inflation persistence has been recently emphasized by, among others, Altissimo, Mojon and Zaffaroni (2009), Bils and Klenow (2004), and Carvalho (2006). Price indices are the result of aggregation over sectorial prices set by firms that are *heterogeneous* along some dimensions. If the degree of heterogeneity is non-negligible, if the dynamic process that drives sectorial prices is characterized by an appreciable degree of temporal dependence, and if a sufficient number of units display large persistence, then the aggregate process will have an order of integration larger than zero. Moreover, macroeconomic aggregates are typically constructed as the weighted average of cross-sectionally dependent individual units, which are linear in logs and not in levels: the aggregate variable is then the sum of multiplicative processes, and is a nonlinear function of its components even when they are added linearly. If both elements are jointly taken into account, proper aggregation leads to a long memory process characterized by a highly nonlinear pattern, which may behave very differently from both linear AR and ARFIMA models (which are nested as special cases). The dynamic properties of such a process are captured by an ACF with changing concavity, which denotes the presence of long and asymmetric cycles (Abadir and Talmain, 2002).

Our paper builds on these contributions and provide a framework to investigate inflation dynamics. Our results suggest that inflation dynamics in 20 OECD countries is consistent with a flexible long memory and nonlinear process but not with linear ARMA processes.

The rest of our paper is organized as follows. Section 2 discusses the rationale of why a nonlinear and long memory process is likely to be a more appropriate framework than linear autoregressive processes to model inflation dynamics. In Section 3 we estimate the sample inflation ACFs for 20 OECD countries over a period spanning five decades, and assess the relative performance of a standard AR(p) model compared with that of an aggregate process derived from a general equilibrium model with heterogenous agents. Section 4 concludes.

2 Long memory and nonlinearities in aggregate inflation

Suppose X_{it} is a time series whose logarithm follows an AR(1) process:

$$x_{it} = \theta_i x_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim i.i.d.N\left(0, \sigma_i^2\right) \tag{1}$$

where $x_{it} = \ln(X_{it})$, $|\theta_i| \sim \beta eta\left(\bar{\theta}, \sigma_\theta^2\right)$, with $i = 1, \ldots, N$, and $t = 1, ..., T$. If $|\theta_i| < 1$, the ACF of x_{it} is strictly convex and decays to zero at an exponential rate, with speed of convergence inversely proportional to $|\theta_i|$.

Consider now the aggregate process

$$x_t \equiv \sum_{i=1}^{N} h_i x_{it}, \tag{2}$$

where h_i denotes the weight attached to unit i, with $\sum_{i=1}^{N} h_i = 1$. If the individual units are uncorrelated, sufficiently persistent and heterogeneous (i.e. there is a sufficiently large number of θ_i close to one and a large σ_θ^2), x_t is a long memory process whose autocorrelation function is strictly convex and decays to zero at a hyperbolic rate if N is large.

However, in a macroeconomic setting, two further circumstances must be taken into account. First, some form of cross-correlation among the individual units must be accounted for, i.e. $E(\varepsilon_{it}\varepsilon_{jt}) \neq 0$ for some $i \neq j$. Second, macroeconomic variables are typically constructed by summing up not the logarithms but the levels of the individual units X_{it}. The aggregate process is then given by

$$\tilde{x}_t \equiv \ln\left[\sum_{i=1}^{N} h_i \exp(x_{it})\right] \neq x_t \equiv \sum_{i=1}^{N} h_i x_{it}. \tag{3}$$

Unlike x_t, $\tilde{x}_t$ is a highly nonlinear function of the individual x_{it} and its dynamic properties can be substantially different from those of x_t.

Abadir and Talmain (2002) show that, for a process like (3), use of leading term approximation gives an autocorrelation function with a hyperbolic rate of decay, which is typical of long memory processes, and that displays, unlike ARFIMA models, changing concavity. Smaller order terms of the series expansion have alternate signs, which add cyclical deviations to the ACF captured by a trigonometric function. The exact ACF functional form reads as follows:

$$\tilde{\rho}_\tau^{AGG} = \frac{1 - a\left[1 - \cos(\omega\tau)\right]}{1 + b\tau^c} \tag{4}$$

where the parameter c measures the rate-of-decay of shocks, b regulates the "on impact" slope of the ACF, a influences the impact of the oscillations implied by the presence of the cosine function in the numerator, and ω drives the frequency of such oscillations (see Abadir, Caggiano and Talmain, 2008).

3 Inflation dynamics in OECD countries

We use quarterly data from 1960 to 2006 on Consumer Price Index (CPI), obtained from the OECD Main Economic Indicators, for 20 economies to calculate the inflation rate $\pi_{j,t} = 100\left[(P_{j,t} - P_{j,t-4})/P_{j,t-4}\right]$, where $P_{j,t}$ is seasonally adjusted CPI for country j. We calculate year-on-year inflation on the basis that this is the measure of inflation targeted by central banks.

We first estimate the sample ACF of inflation for each country and select the number of significant lags using the stationary bootstrap (Politis and Romano, 1994).[1] We then estimate by Nonlinear Least Squares:

$$\tilde{\rho}^{j}_{\tau} = \rho_{k,\tau} + \varepsilon^{j}_{\tau},\ \tau = 1, \ldots, \tau^{*} \tag{5}$$

where $\tilde{\rho}^{j}_{\tau}$ is the sample autocorrelation at lag τ for country j, and ρ_k denotes the theoretical ACF implied by model k, where $k \in \{AR, AGG\}$.

Figure 1 plots the sample and fitted ACFs. First, in all cases the AGG process replicates the sample ACFs much better than the competing AR model. Second, often the AR model is forced to deliver the wrong sign of the concavity of the ACF at low lags to replicate the behaviour of the sample ACF in the middle of the sample. As a consequence, it tends to overestimate the ACF at low lags, and hence the rate of decay of shocks. The computation of the implied Sum of the Autoregressive Coefficients returns an average value of 0.99, with a minimum of 0.935 for Austria and a maximum of 0.999 for Switzerland. This suggests that, *if* the true DGP were a linear autoregressive process, aggregate inflation would be a unit root, or a near unit root process. Finally, in some cases the estimated autoregressive processes deliver implausible high-frequency oscillations.

Table 1 reports two measures of goodness of fit. The first is the Schwarz Criterion (SC). In all cases, the SC is minimized when the sample ACF is fitted by (4). Since

[1] To save space, confidence bands are not reported. The computation of the 90% confidence bands, which confirms the significance of basically all lags of our ACFs in Figure 1, is provided by Caggiano and Castelnuovo (2008), along with a discussion on the robustness of our results to heavy-tailed empirical distributions. See also Caggiano and Leonida (2009) for details on the use of the stationary bootstrap for constructing confidence bands for sample ACFs.

the SC is designed for nested models selection, we also provide a second measure of goodness of fit, the Q-ratio (see Cogley and Nason, 1995). Let

$$Q_k^j = \frac{1}{\tau^*} \sum\nolimits_{\tau=1}^{\tau^*} \left(\widehat{\rho}_{k,\tau}^j - \tilde{\rho}_\tau^j \right)^2, \quad k \in \{AR, AGG\} \tag{6}$$

be the Q-statistic for model k and country j, where $\tilde{\rho}_\tau^j$ is the lag-τ sample ACF for country j, and $\widehat{\rho}_{k,\tau}^j$ is the model-k fitted ACF, and τ^* is number of significant lags. The Q-ratio for country j is then given by:

$$Q_r^j = Q_{AR}^j / Q_{AGG}^j. \tag{7}$$

A value of Q_r larger than one indicates a better fit of the aggregate process relative to the AR(p).

The values reported in Table 1 confirm the that the aggregate process (4) replicates the sample ACFs much better than the AR(p). These results are in line with the claim by Caballero and Engel (2003), i.e. estimates of persistence based on partial-adjustment ARMA models are likely to be incorrect.

4 Conclusions

Our empirical analysis shows that i) the dynamics of inflation is characterized by long-lasting asymmetric fluctuations that tend to slowly fade away; ii) a parsimonious statistical process, implied by models with heterogenous agents, replicates well the observed inflation dynamics and improves substantially upon a standard AR(p) process across all countries included in our dataset. These findings bring further support to the need of explicitly modeling heterogeneity in macroeconomic models.

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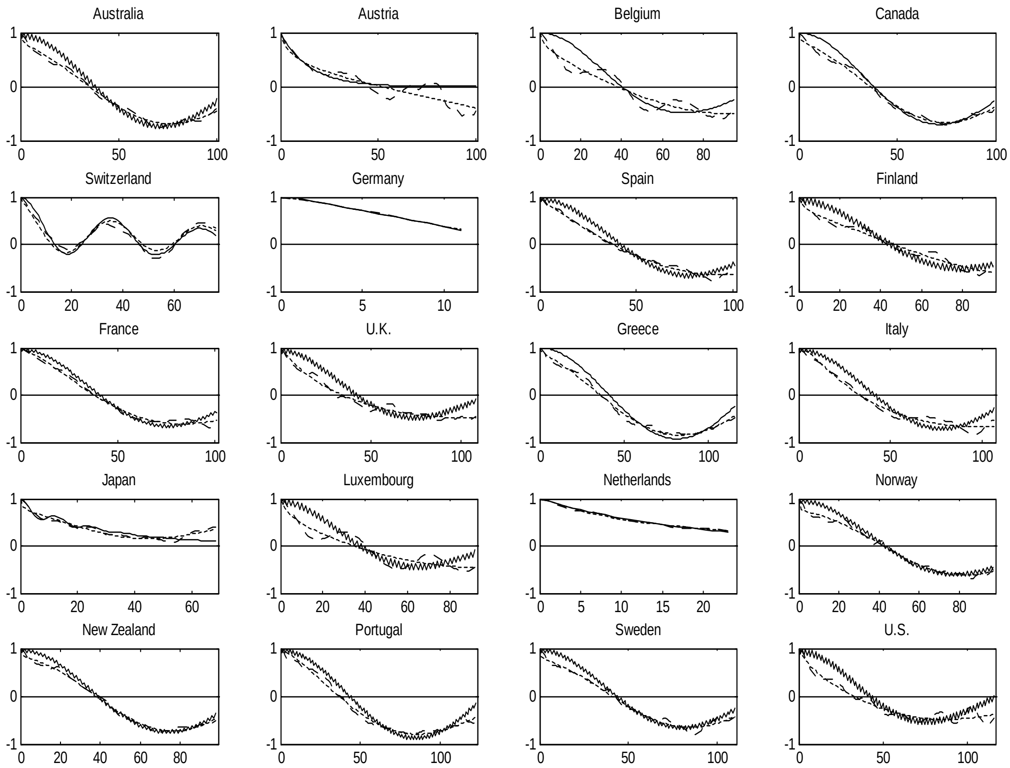

Figure 1: **SAMPLE AND FITTED ACFs: FULL SAMPLE.** Dashed line: Sample ACF. Dotted line: ACT-fitted ACF. Solid line: AR-fitted ACF. Lags selected according to the criterion explained in the text.

j	τ^*	$\frac{SC}{AGG}$	$\frac{SC}{AR(p)}$	Q_{AGG}	Q_r	j	τ^*	$\frac{SC}{AGG}$	$\frac{SC}{AR(p)}$	Q_{AGG}	Q_r
AS	101	-5.79	-3.69(3)	0.25	8.55	GR	117	-5.72	-3.66(2)	0.28	8.51
AT	101	-3.94	-3.04(1)	1.59	2.84	IT	107	-4.95	-3.10(3)	0.59	6.64
BE	96	-3.97	-2.84(2)	1.53	3.44	JP	69	-5.73	-4.30(3)	0.25	4.47
CA	100	-5.62	-4.01(2)	0.30	5.44	LX	93	-3.69	-2.65(3)	2.04	2.93
SZ	77	-4.57	-4.50(3)	0.81	1.14	NL	24	-7.59	-7.20(1)	0.03	2.18
GE	12	-10.44	-8.76(4)	0.00	4.38	NO	98	-5.39	-3.85(3)	0.37	4.88
SP	102	-5.94	-3.56(3)	0.22	11.35	NZ	99	-5.91	-4.39(3)	0.22	4.81
FI	96	-4.79	-3.24(3)	0.68	4.91	PO	123	-5.54	-3.52(3)	0.33	7.73
FR	102	-5.73	-4.23(3)	0.27	4.68	SW	111	-5.50	-3.82(3)	0.34	5.60
UK	109	-5.30	-2.91(3)	0.42	11.32	US	117	-4.62	-2.72(3)	0.82	7.05

Table 1: **SAMPLE AND FITTED ACFs - FULL SAMPLE: GOODNESS OF FIT.** 'j' stands for 'Country'. 'SC' denotes Schwarz Criterion. The number of lags p of the AR(p) processes is reported in brackets. 'Q_{AGG}' is the Q-statistic for the aggregate model of Section 2. 'Qr' is the ratio between Q_{AR} and Q_{AGG}.