Fitting U.S. Trend Inflation: A Rolling-Window Approach*

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Abstract

The role of trend inflation shocks for the U.S. macroeconomic dynamics is investigated by estimating two DSGE models of the business cycle. Policymakers are assumed to be concerned with a time-varying inflation target, which is modeled as a persistent and stochastic process. The identification of trend inflation shocks (as opposed to a number of alternative innovations) is achieved by exploiting the measure of trend inflation recently proposed by Arouba and Schorfheide (2011, *American Economic Journal: Macroeconomics*). Our main findings point to a substantial contribution of trend inflation shocks for the volatility of inflation and the policy rate. Such contribution is found to be time-dependent and highest during the mid-1970s to mid-1980s.

Keywords: Trend inflation shocks, new-Keynesian DSGE models, rolling-window approach, great moderation,.

JEL codes: E31, E32, E52.

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1 Introduction

This paper investigates the following questions: i) Are shocks to trend inflation relevant to describing the U.S. macroeconomic dynamics? ii) Which distortions arise if trend inflation shocks are not modeled? iii) Has the relevance of trend inflation shocks varied over time? iv) What is the relative importance of such shocks with respect to more conventional, temporary monetary policy shocks?

We answer these questions by estimating two different Dynamic Stochastic General Equilibrium (DSGE) models of the business cycle that are commonly employed in the monetary macroeconomic literature. The first one is a small-scale new-Keynesian model featuring a few nominal and real frictions. The presence of nominal rigidities gives monetary policy shocks the power to influence the real side of the economy on top of inflation and the policy rates. A version of this model is extensively analyzed in Woodford (2003). The second model is a version of the medium-scale framework popularized by Christiano, Eichenbaum, and Evans (2005) and taken to the U.S. data by Smets and Wouters (2007). This model features a number of nominal and real frictions in the attempt to offer a richer representation of the shocks affecting the U.S. economy and the transmission mechanisms regulating their impact on the macroeconomic dynamics. We modify these two frameworks by formalizing a time-varying inflation target, otherwise labeled 'trend inflation', which we model as a persistent process whose variance is jointly estimated with the rest of our models' parameters. In this way, shocks to trend inflation concur in determining the volatility of our variables of interest.

Our empirical exercise aims at assessing to what extent trend inflation shocks participated to the formation of the great inflation in the 1970s and the great moderation in the post-1984 period. Hence, the contribution of this paper to the existing literature is twofold. First, we include an exogenous measure of trend inflation in the set of observables employed to estimate our DSGE models. Second, we perform rolling-window estimations, which allow us to compare the evolution of the model parameters, the volatilities of the shocks over time, and the contribution of trend inflation shocks in determining the U.S. macroeconomic dynamics (in particular, the volatilities of inflation, output, and the policy rate).

Two different interpretations may be assigned to the evolution of the low-frequency component of inflation. The first one refers to such movements as changes in the inflation target pursued by the Federal Reserve over time. According to this interpretation, the upward trending inflation rate occurred in the 1970s may be interpreted as *'[...] due to a systematic tendency for Federal Reserve policy to translate the short-run price pressures set off by adverse supply shocks into more persistent changes in the inflation rate itself - part of an effort by policymakers to avoid al least some of the contractionary impact those shocks would otherwise have had on the real economy.'* (Ireland, 2007, p. 1853). Given that, by assumption, agents possess full information on the structure of the economy and compute their expectations rationally, this is our preferred interpretation of trend inflation in this paper. The second interpretation relates to a learning process by the Federal Reserve, which got to understand the inflation-output volatility trade-off in place while observing the reaction of the U.S. economic environment to its policy moves. This interpretation suggests that the *'[...] changing beliefs about the output-inflation trade-off generated a pronounced low-frequency, hump-shaped pattern in inflation.'* (Cogley, Primiceri, and Sargent, 2010, p. 57). Following Ireland (2007) and a number of subsequent contributions, we model trend inflation as an exogenous, autoregressive process taking care of the evolution of the Federal Reserve's inflation target.[1] Hence, we formalize neither the decisional process by the Federal Reserve to vary its target over time nor the learning process possibly inducing the evolution of such target. Hence, our contribution provides a quantitative assessment on the relevance of the shocks hitting the low-frequency component of inflation, and leaves some related modeling challenges to future research.

Our results read as follows. First and foremost, a substantial participation of trend inflation shocks to the volatilities of inflation and the policy rate is detected. An investigation involving the sample 1965-2005 suggests that trend inflation shocks are the main determinant of the volatility of inflation and the federal funds rate, and are relatively more important than

[1]See Castelnuovo (2012c) for an attempt to endogenize the Federal Reserve's inflation target on the basis of past inflation realizations.

standard monetary policy shocks. Second, such shocks explain a negligible portion of the volatility of the U.S. output. Third, the omission of trend inflation shocks leads to an overestimation of the role of supply shocks in determining the dynamics of inflation and the federal funds rate. In particular, the estimated contribution of wage mark-up shocks turns out to be doubled. Fourth, there are evident instabilities in the estimated volatility of trend inflation shocks as well as the other structural shocks modeled in our analysis. Fifth, the contribution of trend inflation shocks to inflation and the policy rate dynamics is found to be time-varying. Sixth, even when allowing for parameter instability in our estimated frameworks, trend inflation shocks emerge as extremely important to explain the evolution of the nominal side of the U.S. economy. In particular, their share is large when the mid-1970s to mid-1980s are considered. Finally, the relative importance of trend inflation shocks in explaining the dynamics of the nominal (real) side of the economy is larger (smaller) than that of standard, temporary policy shocks.

Movements in trend inflation have already been identified as one of the possible drivers of the post-WWII U.S. macroeconomic environment.[2] According to Cogley, Primiceri, and Sargent (2010), trend inflation is the single most important factor behind the U.S. inflation dynamics. Evidence in favor of a drop in the persistence of the inflation gap correlated with a fall in trend inflation is provided by Cogley and Sbordone (2008). Coibion and Gorodnichenko (2011a) couple a small-scale DSGE model with a policy rule featuring a time-varying inflation target, and show that the reduction in trend inflation occurred since the mid-1980s has importantly contributed to lead the U.S. economy to the Great Moderation phase.[3]

Our paper makes further steps in the assessment of the relevance of trend

[2]Evidence in favor of trend inflation's variability is provided, among others, by Belaygorod and Dueker (2005), Cogley and Sargent (2005b), Kozicki and Tinsley (2005), Ireland (2007), Stock and Watson (2007), Cogley and Sbordone (2008), Leigh (2008), Kozicki and Tinsley (2009), Sbordone, Tambalotti, Rao, and Walsh (2010), Castelnuovo (2010), Coibion and Gorodnichenko (2011a), Coibion and Gorodnichenko (2011b), Aruoba and Schorfheide (2011), Del Negro and Eusepi (2011), Castelnuovo, Greco, and Raggi (2012), Castelnuovo (2012c).

[3]Further elaborations by Ascari, Branzoli, and Castelnuovo (2011) unveil an interaction involving trend inflation and wage indexation that turns out to be relevant in assessing policymakers' ability to anchor inflation expectations in the U.S. economic environment.

inflation as for post-WWII U.S. inflation dynamics. In particular, we conduct rolling-window estimations of a small-scale DSGE model featuring trend inflation and temporary policy shocks with post-WWII U.S. data. To do so, we employ a set of macroeconomic indicators, among which the empirical proxy for trend inflation recently proposed by Aruoba and Schorfheide (2011).[4] This is, in our opinion, a crucial departure from the existing literature. Several reasons justify the use of an 'observable' measure of trend inflation. First, trend inflation is often interpreted as the inflation target pursued by the Federal Reserve.[5] Hence, it is typically modeled as a latent process entering a simple policy rule. This is problematic from an econometric standpoint, in that two persistent latent processes (the trend inflation process and the monetary policy shock process) enter the (log-linearized) policy rule jointly. Then, it becomes difficult to disentangle the effects of these two shocks on the endogenous variables of interest. The employment of a measure of trend inflation, on top of a set of 'standard' macroeconomic indicators (among which we include the federal funds rate), allows us to circumvent this identification issue and sharpen our estimates on the effects of trend inflation against other shocks. Second, as shown by Castelnuovo, Greco, and Raggi (2012), there is a huge amount of heterogeneity as for the estimates of trend inflation in the literature. Such heterogeneity is likely to be due to differences in cross-equation restrictions, assumptions on expectation formation, observables used in the estimation, and a variety of other factors. The employment of a proxy for trend inflation computed 'externally' to our DSGE models makes trend inflation robust to model misspecification. Third, different sample choices may give rise to different estimates of trend inflation for the very same point in time due to, e.g., sampling uncertainty.

[4] Aruoba and Schorfheide (2011) consider survey measures (1-year- and 10-year-ahead inflation expectations coming from the Survey of Professional Forecasters) and a low-frequency component of GDP deflator inflation extracted with a Band-Pass filter. The common factor extracted by combining such "observables" with a small state-space model is their empirical proxy for trend inflation.

[5] Imperfect knowledge of the economic structure and the evolution of the perceived inflation-output volatility trade-off by the Federal Reserve is one of the possible ways to make the trend inflation process endogenous (in this paper, it is assumed to be an exogenous process). Interesting efforts in this direction have already been undertaken by Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006), Carboni and Ellison (2009), and Milani (2009).

This is clearly an unfortunate side-effect of estimations conducted without proxies for trend inflation, above all when attempting to assess its role over time. The employment of an 'observable' for trend inflation help us tackling these issues.

We proceed as follows. First, we estimate our DSGE models over the post-WWII sample 1965-2005.[6] Conditional on the estimated framework, we compute the variance decomposition of the observables employed in our Bayesian estimation. This allows us to assess the relative role of trend inflation shocks, standard transitory monetary policy shocks, and other identified structural innovations. As anticipated, however, our exercise is designed to detect the possibly *time-varying* role played by trend inflation shocks in affecting the post-WWII U.S. macroeconomic dynamics. One may think of the turbulent 1970s as being a very different environment for policymakers when contrasted with the 'great moderation' phase. We tackle this issue by running rolling-window estimations of our DSGE model with Bayesian techniques, a strategy already followed (in models featuring no trend inflation) by Canova (2009), Giacomini and Rossi (2010), Cantore, Ferroni, and Léon-Ledesma (2011), Canova and Ferroni (2012), and Castelnuovo (2012a). This strategy enables us to detect instabilities in possibly all structural parameters of our framework by relying on standard techniques suited to estimate linear frameworks.

Different approaches to model instabilities in the structural parameters are available to econometricians. Modern monetary DSGE models of the business cycle that features time-varying coefficients and stochastic volatilities have been estimated by Fernández-Villaverde and Rubio-Ramírez (2007 a,b) and Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010). Time-varying volatilities are assumed to follow an AR(1) process in log-terms to ensure the positivity of such volatilities. This leads to a mix of *levels* (of the structural shocks) and *logs* (of the volatilities of the structural shocks) that creates a non-linear structure. A second-order approximation of the policy functions is therefore needed to capture such relevant non-linearities and estimate the parameters of interest. This has two consequences. First, ratio-

[6]The sample width is due to our willingness to use the original estimates of trend inflation by Aruoba and Schorfheide (2011).

nal expectations are solved by appealing to perturbation methods (that are superior to alternatives, see Fernández-Villaverde and Rubio-Ramírez (2010) and the references therein). Second, once the model is solved for rational expectations, the likelihood function is evaluated by appealing to particle filtering. This methodology is extremely powerful and econometrically clean. However, given its computations costs, it forces the econometrician to stick to a limited number of time-varying parameters. This is unfortunate, because given the likely covariance of the structural parameters of interest, fixing a sub-set of parameters while leaving the complementary sub-set free to change is likely to induce biases in the simulated density (see Canova and Ferroni (2012) for a discussion). In contrast, rolling-window estimations allow to trace instabilities in (possibly) *all* structural parameters in a convenient manner, so enabling us to overcome this issue.

Another alternative to handle parameter instability is represented by regime-switching, which has recently been adopted to estimate DSGE frameworks by a number of authors (Bianchi (2011), Liu, Waggoner, and Zha (2011)). This technique is obviously extremely powerful, in that it enables to identify different phases characterizing the economic environment (e.g., 'tranquil' times as opposed to 'turbulent' periods). As a matter of fact, however, it forces the data to 'pick and choose' among a necessarily limited number of states, i.e., to 'discretize' the economy. Differently, rolling-windows, in principle, just 'let the data speak' as for the relevant 'state' among the possible infinite ones per each given window, so offering a natural generalization of the regime-switching approach.

Wrapping up, we see the rolling-window methodology as complementary to the time-varying coefficients/stochastic-volatility and the regime-switching approaches. Of course, there is no free lunch. The cost is that of abstracting from the role that time-varying parameters may play in influencing agents' expectations, i.e., with our rolling-window approach we are forced to assume that agents have neither memory of the past windows nor are able to exploit past and current information on parameters' drifts to form expectations on the future evolution of the economy. Moreover, rolling-window estimation is adequate as an initial step in detecting instabilities in the structural parameters of their framework. They, however, do not represent a generalization of the alternatives we discusses earlier such as regime-switching coefficients or smoothly time-varying coefficients and volatilities. In light of the results presented in this paper, we see the application of more sophisticated techniques to detect structural instabilities in a DSGE framework as the natural following step, which we leave to future research.

The paper closest to ours are probably Ireland (2007) and Cogley, Primiceri, and Sargent (2010) as for the small-scale model application, and Sbordone, Tambalotti, Rao, and Walsh (2010) and Del Negro and Eusepi (2011) as regards the application with the Smets and Wouters' (2007) model. Ireland (2007) builds up a small-scale DSGE model in which firms may index their prices to trend inflation, which is interpreted as the inflation target pursued by the Federal Reserve. He finds that trend inflation is responsible for the bulk of the volatility of inflation and the policy rate in the sample under scrutiny. Ireland's (2007) analysis assumes the structure of the U.S. economy not to feature any instability over the post-WWII sample. Cogley, Primiceri, and Sargent (2010) relax this assumption by engaging in a subsample analysis focusing on the pre- vs. post-Volcker periods. They document a fall in the inflation gap persistence and volatility, which is shown to be related to the reduction in the variance of trend inflation shocks during the great moderation. Sbordone, Tambalotti, Rao, and Walsh (2010) estimate a medium-scale model à la Smets and Wouters (2007) conditional on the great moderation sample. Their evidence supports the relevance of trend inflation shocks as for the volatility of the nominal side of the economy. Del Negro and Eusepi (2011) estimate a variety of medium-scale models by employing Survey of Professional Forecaster's short-term inflation expectations measures on top of a standard set of macroeconomic indicators. In particular, they deal with a medium-scale model à la Smets and Wouters (2007) that features a fix inflation target; the same-medium scale model but with a time-varying inflation target; and an imperfect information model in which agents infer the time-varying target from the observation of the federal funds rate as in Erceg and Levin (2003). Their empirical exercise points to the empirical superiority of the perfect information model with the time-varying trend inflation. As in this papers, we assume trend inflation to follow an autoregressive process whose variance is estimated jointly with the rest of the parameters of interest. We also find support for the role of trend inflation shocks as drivers of the U.S. inflation and policy rate in the United States. We add on these literature by showing that the contribution of such shocks in determining the volatility of the nominal side of the economy has followed an 'inverted U-shape', with the highest value, at least as for inflation, recorded in the mid-1980s. Moreover, we show that models omitting trend inflation shocks overestimate the contribution of supply shocks in determining inflation and the federal funds rate. In particular, the shocks to the wage mark-up are those that get inflated the most in presence of this form of model misspecification.

This paper develops as follows. Section 2 presents a helicopter tour over the literature dealing with the estimation of the trend inflation process. Section 3 presents our small-scale model, its estimates, and the variance decomposition analysis performed with such framework. Section 4 develops our rolling-window investigation conditional on the small-scale model presented in Section 3. Section 5 moves to the analysis based on the Smets and Wouters (2007) model, and presents our full-sample and rolling-window estimates. Section 6 concludes. An Appendix including some details on the Bayesian estimation of our DSGE models as well as the description of the Smets and Wouters (2007) framework is also provided.

2 Trend inflation estimates: A helicopter tour

Figure 1 displays a number of trend inflation estimates in the literature. It focuses on a selection of contributions in the literature, i.e., Kozicki and Tinsley (2005), Ireland (2007), Kozicki and Tinsley (2009), Coibion and Gorodnichenko (2011), and Aruoba and Schorfheide (2011), and Castelnuovo, Greco, and Raggi (2012).[7] To offer a sense of how such estimates may

[7] The sources of these estimates read as follows. Kozicki and Tinsley (2005 and 2009) and Ireland (2007): Original files provided by the authors. Coibion and Gordnichenko (2011): American Economic Review (website), their paper, zip file under "Additional Materials Download Data Set", "GreenBookForecasts for AER.xlsx" file, Trend Inflation, Smoothed estimates. Monthly estimates converted to quarterly estimates by selecting the latest available observation within each quarter. Aruoba and Schorfheide (2011): American Economic Review (website), their paper, zip file under "Additional Materials - Download Data Set", "inflation-target.xls" file, "filtered f2" estimates. Castelnuovo,

relate to a measure of inflation, we contrast them with the U.S. GDP deflator inflation rate.[8]

Panel [1,1] displays Kozicki and Tinsley's (2005) estimate of trend inflation. Kozicki and Tinsley (2005) employ a VAR model featuring variations in the Fed's inflation target that are imperfectly perceived by the private sector, which is unable to perfectly distinguish between permanent target shocks and transitory policy shocks and learn over this difference as more information enter its information set. The changes of the inflation target partly reflect the Fed's response to supply shocks hitting the U.S. economy over the post-WWII period. Kozicki and Tinsley's (2005) estimated target moves from values smaller than 2% in the early 1960s to values close to 8% at the end of the 1970s. Their trend inflation measure takes somewhat higher values than ours in the late 1960 and early 1970s. Interestingly, their estimated inflation target drops to zero during the Volcker disinflation, then it gradually returns to around 4% from the mid-1980s to the mid-1990s, and slightly lowers later on.

Ireland (2007) estimates a microfounded DSGE model of the business cycle with perfect information enjoyed by all agents of the economic system. The trend inflation process is modeled as a random walk.[9] His estimate of the inflation target is quite similar to the one proposed by Stock and Watson (2007), who work with reduced-form models for the U.S. inflation rate that allow for trend inflation and time-varying volatility of the stochastic components they consider, and it is statistically consistent with the one proposed by Cogley and Sbordone (2008), once the uncertainty surrounding the latter is taken into account. Panel [1,2] superimposes Ireland's (2007) estimate of the Federal Reserve's target to actual inflation. Ireland's (2007) trend infla-

Greco and Raggi (2011): Files available upon request.

[8] Note that just part of the authors whose estimates are analyzed in this Section work with GDP deflator inflation. Therefore, the difference between inflation and a given trend inflation estimate represents by no means an attempt to "judge" its "plausibility". Again, the presence of trend inflation is intended to offer a sense of the economic situation in place in different phases of the post-WWII U.S. economic history.

[9] Ireland (2007) contrasts different processes of trend inflation, some of which allow for a systematic reaction to structural shocks hitting the economic system. The role of such shocks, however, turns out to be empirically negligible. Panel [1,2] shows the case labelled by Ireland (2007, Figure 5, page 1869) as "Federal Reserve's Target as Implied by the Constrained Model with an Exogenous Inflation Target".

tion estimate 'filters' the U.S. inflation rate and captures its low-frequency component. He obtains his estimate by working with a microfounded DSGE model that features firms that, when not reoptimizing their prices, just set them conditional on a convex combination involving past and trend inflation. As in our paper, the central bank is assumed to react to a measure of the inflation gap conditional on a time-varying inflation target.

A different picture emerges from Kozicki and Tinsley's (2009) contribution. They estimate a policy rule with time-varying coefficients using real-time Greenbook data and imposing a set of restrictions consistent with intermediate money supply targeting. The resulting trend inflation is estimated to lie between 6.1 and 7.2% in the period 1970-1980, then dramatically falls to about 3% in the 1980s and 1990s, a phase in which no intermediate money supply targeting is implemented.[10] Panel [2,1] shows a dramatic difference between their target and, say, Ireland's (2007). Such difference may be due, among other reasons, to the fact that they account for some restrictions implied by money supply targeting.

Coibion and Gorodnichenko (2011) propose an estimate of trend inflation conditional on a Taylor rule featuring time-varying coefficients and estimated with real-time Greenbook data. Such rule features a time-varying intercept, which they interpret as being a combination of time-dependent objects such as trend inflation, the equilibrium real interest rate, and the target growth rates for the output growth and the output gap. Some assumptions on the evolution of the last three objects enable Coibion and Gorodnichenko (2011) to recover the evolution of the trend inflation process. Their estimated target, displayed in Panel [2,2], turns out to be smoother than ours, with an empirical standard deviation reading 1.76 vs. the larger 1.92 associated to Ireland's (2007) estimate (conditional on Coibion and Gorodnichenko's sample). However, a similar pattern emerges, in that both measures clearly follow the upward inflation trend of the 1970s, the Volcker disinflation occurred in the early 1980s, and the somewhat gradual stabilization of inflation realized in the 1990s. Notably, the correlation between these two measures of trend

[10]Evidence in favor of a decline of the role of money growth as a driver of the (Hodrick-Prescott filtered) U.S. real GDP when moving from the 1970s to the great moderation is found by Castelnuovo (2012a).

inflation reads 0.84.

Panel [3,2] focuses on the estimate obtained by Castelnuovo, Greco, and Raggi (2012). They work with a very flexible Taylor rule that features possibly time-varying policy coefficients, trend inflation, and heteroskedastic shocks. They find clear evidence in favor of such model of the U.S. policy conduct when contrasted with a constrained version of it featuring no time-varying inflation target. Interestingly, their trend inflation estimate, which is obtained with a single-equation approach allowing for policy time-dependence along the previously mentioned dimensions, is extremely similar to Ireland's (2007), whose estimation is obtained with a microfounded DSGE model featuring absence of any time-dependent object.

Finally, a different approach to compute trend inflation is followed by Aruoba and Schorfheide (2011). They combine three different measures of inflation - quarterly GDP inflation filtered through a one-sided band pass filter, one-year- and ten-year-ahead inflation expectations from the Survey of Professional Forecasters - by using a small state-space model. Then, they extract the common factor via the Kalman filter. Panel [3,3] plots Aruoba and Schorfheide's (2011) estimate. Such measure turns out to be quite similar to those proposed by Ireland (2007) and Castelnuovo, Greco, and Raggi (2012). This is interesting, in light of the fact that Aruoba and Schorfheide's estimate involves measures of expectations that are not considered in the other two investigations.

Figure 1 points to the heterogeneity of estimates present in the literature. In summary, Ireland's (2007), Castelnuovo et al (2011) and Aruoba and Schorfheide's (2011) propose extremely similar estimates (despite of the strikingly different methodologies, samples, and data employed). Coibion and Gorodnichenko's (2011) estimate is fairly similar to those obtained by these three papers. Kozicki and Tinsley (2005) offer a somewhat different picture, in that their trend inflation estimate suggest realizations like the dramatic drop to zero in the early 1980s, which are absent in the remaining estimate under scrutiny. Finally, Kozicki and Tinsley's (2009) suggest a quite stable trend inflation estimate that features a clear 'break in mean' at the end of the 1970s. In drawing these comparisons, one must keep in mind that differences may be due to a variety of reasons, including differences in selected samples, data transformation (e.g., the quarterly inflation rate used in our exercise and in Ireland, 2007, as opposed to the four-quarter inflation rate employed by Kozicki and Tinsley, 2005), structure imposed to the data (e.g., simple rules as in our case, Kozicki and Tinsley, 2005, and Coibion and Gorodnichenko, 2011, structural vector autoregressions as in Kozicki and Tinsley, 2005, state-space representations as in Aruoba and Schorfheide, 2011, DSGE frameworks as in Ireland, 2007), vintage of the data ('real-time' vs. 'revised' data).

In our DSGE model-based analysis, we will focus on Aruoba and Schorfheide's (2011) estimate on trend inflation for various reasons. First, it is a model-free measure of trend inflation. DSGE models are typically misspecified along some dimensions (Del Negro, Schorfheide, Smets, and Wouters (2007)). Clearly, model misspecification may lead to a potentially distorted estimate of the trend inflation process. From this standpoint, it is 'safer' to rely on externally combined observables when feeding the measurement equations of our DSGE models with a measure of trend inflation. Second, expectations over future inflation are likely to be quite informative on the trend inflation process, in that rational agents should form their expectations over future inflation by appealing, first and foremost, to their predictions on the low-frequency component of inflation. Third, as already stressed in the Introduction, the role of innovations to the policy rate *per se* vs. innovations to the inflation target is hard to identify unless an empirical proxy of trend inflation is employed.

We now turn to our structural analysis.

3 A structural analysis with a small-scale DSGE model

The small-scale framework we work with reads as follows:

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\begin{aligned}
\pi_t &= \beta(1+\alpha\beta)^{-1}E_t\pi_{t+1} + \alpha(1+\alpha\beta)^{-1}\pi_{t-1} + \kappa x_t + \varepsilon_t^{\pi}, & (1)\\
x_t &= \gamma E_t x_{t+1} + (1-\gamma)x_{t-1} - \sigma^{-1}(R_t - E_t\pi_{t+1}) + \varepsilon_t^{x}, & (2)\\
R_t &= (1-\phi_R)(\pi_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_x x_t) + \phi_R R_{t-1} + \varepsilon_t^{R}, & (3)\\
\pi_t^* &= \rho_* \pi_{t-1}^* + v_t^*, & (4)\\
\varepsilon_t^k &= \rho_k \varepsilon_{t-1}^k + v_t^k, k \in \{\pi, x, R\} & (5)\\
v_t^j &\sim i.i.d.N(0, \sigma_j^2), j \in \{\pi, *, x, R\}\,. & (6)
\end{aligned}
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Eq. (1) is an expectational new-Keynesian Phillips curve (NKPC) in which π_t stands for the inflation rate, β represents the discount factor, x_t identifies the output gap, whose impact on current inflation is influenced by the slope-parameter κ, α identifies indexation to past inflation, and ε_t^{π} may be interpreted as a 'cost-push shock'; γ is the weight of the forward-looking component in the intertemporal IS curve (2); σ^{-1} stands for households' intertemporal elasticity of substitution; ε_t^{x} is a stochastic component that works as a 'demand' shock; ϕ_π, ϕ_y, and ϕ_R are the policy parameters capturing the systematic monetary part of the monetary policy conduct, which is here represented by a standard Taylor rule (3); the monetary policy shock ε_t^{R} allows for a stochastic evolution of the policy rate.

The evolution of the inflation target - formalized by eq. (4) - is dictated by the autoregressive parameter ρ_* as well as the volatility σ_* of its innovation ε_t^*. This process is typically assumed to be a random walk or a very-persistent variance-stationary process capturing the low-frequency component of the inflation rate, which are likely to be sensible approximations of the time-varying target set by monetary-policy authorities (see Ireland (2007), Cogley, Primiceri, and Sargent (2010), Coibion and Gorodnichenko (2011a), and Aruoba and Schorfheide (2011), among others). The autoregressive processes (5) are intended to account for the possible persistence of the shocks affecting the economic environment. Such shocks are loaded by the mutually independent martingale-differences (6), which close the model.[11]

[11] As a matter of fact, the shocks to inflation ε_t^{π} and output ε_t^{x} are likely to be convolutions of "deep" innovations. For instance, the "cost-push" shock ε_t^{π} might very well capture price-mark up shocks, shocks to the possibly time-varying elasticity of substitu-

Since the seminar contribution by Clarida, Galí, and Gertler (2000), this model has been extensively employed to scrutinize the drivers of the U.S. macroeconomic dynamics.

The version of the model we consider omits to account for the role played by price dispersion as for the structure of the NKPC in a world in which steady-state trend inflation (in net terms) is allowed to take a strictly positive value. However, a recent paper by Ascari, Castelnuovo, and Rossi (2011) shows that such role is, from an empirical standpoint, likely to be negligible.

3.1 Model estimation

We estimate the model (1)-(6) with Bayesian methods. Bayesian methods have been shown to perform relatively better than alternatives like classical maximum likelihood or GMM as for DSGE models like those employed in this paper (Canova and Sala (2009)). Readers interested into in depth-discussions on the pros and cons of using Bayesian techniques vs. other estimation methods are referred to An and Schorfheide (2007) and Fernández-Villaverde (2010).

3.2 Data and priors

We work with quarterly U.S. data. We employ four observables, which we demean prior to estimation. The output gap is computed as log-deviation of the real GDP with respect to the potential output estimated by the Congressional Budget Office. The inflation rate is the quarterly growth rate of the GDP deflator. For the short-term nominal interest rate we consider the effective federal funds rate expressed in quarterly terms (averages of monthly values). The source of these data is the Federal Reserve Bank of St. Louis' website. As discussed in the introduction, we also employ the measure of trend inflation elaborated by Aruoba and Schorfheide (2011). From the private sector's

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tion among goods, and other disturbances. The same holds as for the "non-policy demand shifter" ε_t^x, which does not allow us to discriminate among (say) investment-specific technology shocks, shocks to consumers' preferences, or fiscal shocks, and others. However, this paper's ultimate goal is to pin down the relative role played by identified shocks such as shocks to trend inflation and monetary policy shocks in shaping the dynamics of the economy. Therefore, the "reduced form" nature of the inflation and output shocks does not prevent us in any manner from performing such an assessment.

standpoint, such measure can be interpreted as an anchor for long-term inflation expectations. Aruoba and Schorfheide's trend inflation estimate is based on three different measures of inflation expectations, i.e., the GDP deflation inflation filtered by a one-sided Band-Pass filter, the one-year ahead inflation expectations provided by the Survey of Professional Forecasters, and the ten-year ahead inflation expectations coming from the same source. Such measures are combined by the employment of a small state-space model with which the common factor is extracted via the Kalman filter.[12] We work with the sample 1965:I-2005:IV.[13]

The priors employed in our estimation are indicated in Table 1. Some parameters are hardly identified in our model, then we calibrate them as it is customary in this literature. The discount factor is fixed to 0.99, a value quite common in this literature. The persistence of the trend inflation model is also calibrated. This is a choice done to avoid having troubles in converging to the ergodic posterior density of our model, troubles that might arise in dealing with this close-to-unit root trend inflation observable. Following Cogley, Primiceri, and Sargent (2010), we set ρ_* to 0.995. This value implies a bounded value for the trend inflation variance in population and, at the same time, it allows us to capture the extremely high persistence of the trend inflation process. An alternative would be to allow for a unit-root in trend inflation. However, Cogley, Primiceri, and Sargent (2010) show that a unit root in trend inflation is likely to induce a very low predictability of the inflation gap by models like the ours. This would be at odds with the fact documented by Cogley, Primiceri, and Sargent (2010), who employ VARs with time-varying coefficients and stochastic volatility to model the post-WWII U.S. data and find that such predictability has been high before the advent of Paul Volcker as Federal Reserve's chairman.

[12] Further details on the construction of this measure of trend inflation are provided in Aruoba and Schorfheide (2011), pp. 70-71.

[13] Aruoba and Schorfheide (2011) employ the sample 1965:I-2005:I. In our rolling-window estimation, we will use windows of fixed-length (details are provided in the following Section). Given the selection of our window-size as well as our set of initial observations (one per each window), we work with a slightly extended sample, i.e., 1965:I-2005:IV. The last three 'observations' of the inflation target we employ in our estimations are obtained by assuming a fixed target during the 2005. Given that such year clearly belongs to the Great Moderation sample, we believe this assumption to be quite plausible.

As for the estimated parameters, we employ standard priors. In particular, we assume a fairly conservative value for the slope of the NKPC, an aggressive reaction to the inflation gap by the Federal Reserve, and a high persistence of the cost-push shock. We are fairly agnostic as for the remaining parameters, whose a-priori domain is suggested by economic considerations.

Notice that, in conducting our Bayesian estimations, we exclude parametrizations consistent with the absence of an equilibrium or multiple equilibria under rational expectations. The latter case is often advocated when describing the U.S. monetary policy conduct during the 1970s (for the seminal paper in this area, see Clarida, Galí, and Gertler (2000)). Likelihood-based estimations of DSGE models with multiple equilibria, however, require the econometrician to choose a single equilibria out of the many, a choice that is not irrelevant as for the moments implied by the model (Castelnuovo (2012b)).[14] Therefore, our analysis focuses on parametrizations implying a unique equilibrium under rational expectations.

3.3 Posteriors

We verify a smooth convergence towards the posterior density by the graphical analysis elaborated by Brooks and Gelman (1998). A visual inspection of the posterior density confirms the absence of bimodalities and plateaus.[15] The outcome of our Bayesian estimation is reported in Table 1. All parameters assume very conventional values. The slope of the Phillips curve takes a posterior-mean value equal to 0.11, quite in line with the calibration suggested by Ireland (2004). Price indexation is very low, a finding not surprising in light of the very high persistence of the cost-push shock autoregressive process. The weight of the forward-looking component is estimated to be larger than that of past output but clearly smaller than one, a result offering support to the hypothesis of habit formation in consumption elaborated by Furher (2000). The intertemporal elasticity of substitution takes a value in line with a variety of estimates in the literature (Benati and

[14] For a prior-free test of indeterminacy in the U.S. based on GMM techniques, see Castelnuovo and Fanelli (2011).

[15] This part of the analysis, not documented here for the sake of brevity, is available upon request.

Surico (2008), Benati and Surico (2009)). Monetary policy is estimated to exert an aggressive reaction against inflation fluctuations (in deviations with respect to trend inflation), a very mild reaction to the output gap, and a reasonable amount of persistence. Shocks are estimated to be persistent. As for our shocks' standard deviations, that of trend inflation is quite precisely estimated.

Are our posterior densities heavily influenced by our choice of employing Aruoba and Schorfheide's (2011) estimate of the Federal Reserve's inflation target? To answer this question, we re-estimated the model without such a proxy, therefore treating trend inflation as an unobservable latent factor. Table 1 (column identified by '(2)') reports our posterior densities. The main change with respect to the baseline scenario regards the standard deviation of the trend inflation process, which turns out to be doubled with respect to the baseline scenario '(1)'. The remaining structural parameters are in general just mildly affected by the omission of the empirical proxy by Aruoba and Schorfheide's (2011). The two parameters affected the most are policymakers' reaction to the output gap, which substantially increases, and the persistence of the cost-push shock, which is basically halved with respect to the baseline case.

Turning to Aruoba and Schorfheide's (2011) proxy again, one may want to dig deeper in order to understand what the role played by short- and long-term inflation expectations is. Aruoba and Schorfheide's estimate of trend inflation is obtained by combining three different measures of inflation expectations, i.e., a low-frequency representation of GDP deflator inflation extracted via a one-sided version of the Band-Pass filter, a 1-year ahead inflation expectation measure, and a 10-year ahead inflation expectation measure. The latter two indicators are from the Survey of Professional Forecasters, which is currently managed by the Federal Reserve Bank of Philadelphia.[16] Survey-based measures may be contaminated by measurement errors. Moreover, they can be misleading indicators of the Federal Reserve's inflation target if expectations are not formed in a fully-rational fashion, or if agents

[16] As for the ten-year ahead inflation expectations, the data for the period 1979-1991 are from the Livingston Survey and the Blue Chip Economic Indicators. All series on inflation expectations come from the Federal Reserve Bank of Philadelphia.

do not have a full knowledge of the structural model in place. To take this 'measurement error' issue into account, we re-estimate the model with a measure of trend inflation exclusively based on the Band-Pass filter as computed by Aruoba and Schorfheide (2011). Table 1 reports our posterior means (see column '(3)'). As a matter of fact, one can hardly notice any variations with respect to the baseline case. The only parameter that seems to be affected by the change in the proxy for trend inflation is price indexation, whose posterior mean turns out to be lower. However, the 90% credible sets suggest that one should be very cautious before claiming a strong effect of the change in our proxy on the estimated value of such parameter. A possible interpretation of this result is that the Band-Pass filtered inflation rate is a good proxy of Aruoba and Schorfheide's (2011) measure of trend inflation. This is hardly surprising, given the correlation between these two measures.[17]

The high degree of persistence of the cost-push shock ε_t^π is due to the high degree of persistence of the inflation rate. In contrast, the degree of price indexation α is estimated to be negligible. However, one may suspect the existence of an alternative mode characterized by a 'high indexation-low cost-push shock persistence', which could emerge conditional on a different set of priors. We investigate this issue by running an alternative estimation conditional on two different a-priori parameter densities, i.e., $\alpha \sim B(0.75, 0.15)$ and $\rho_\pi \sim B(0.25, 0.15)$. The remaining priors are kept as in our 'baseline' case.

Table 1 (column '(4)') collects the outcome of this estimation. Several comments are in order. First, the choice of the priors drive the result as for price indexation and the persistence of the cost-push shock. In particular, the former (posterior mean) reads 0.69, while the latter 0.06. Therefore, different priors may very well 'turn the world upside down' as for these two parameters, which are key to describe the dynamics of the U.S. inflation. Second, the choice of different priors as for these two parameters has got evident implications as far as most of the remaining parameters are concerned.

[17] As documented by Aruoba and Schorfheide in their footnote 7 (2011, p. 71), by regressing their estimates of trend inflation on the measure computed via the Band-Pass filter (1-year ahead inflation expectations / 10-year ahead inflation expectations), one obtains a coefficient equal to 0.57 (0.22 / 0.23).

In particular, the slope of the NKPC gets 'squeezed' towards zero; the degree of forward-lookingness in the IS curve and the persistence of the non-policy demand shock turn out to be substantially lower; the policy reaction to inflation, while remaining aggressive, is estimated to be milder, while that to the output gap much more aggressive; the degree of policy gradualism is found to be higher, while the persistence of the policy shock more moderate; the volatility (standard deviation) of the non-policy demand shock is also found to be larger. Third, and most importantly, the overall 'fit' of the model is measured to be worse conditional on these alternative priors. In terms marginal likelihoods, computed by adopting the 'modified-harmonic mean' approach proposed by Geweke (1999), we found a difference (expressed in log-points) of about 7.5. This translates into a Bayes factor of about 1,808, which provides us with a 'very strong' evidence in favor of the model with 'low indexation and high cost-push shock persistence'.[18] Therefore, in the rest of the paper, we will concentrate on the 'low indexation-high cost-push shock persistence' formulation of the model.

What is the role played by trend inflation shocks as opposed to other innovations in shaping our observables? We investigate this issue in the following Section.

3.4 Variance decomposition

We assess the role of trend inflation shocks by appealing to a standard variance decomposition exercise. Our computations (conditional on the model's posterior mode) will refer to two different horizons, i.e., $8-$step and $\infty-$step ahead. The former one is intended to assess the contribution of the structural shocks at 'business cycle frequencies'. Differently, the second one aims at pinning down the drivers of the U.S. macroeconomic 'unconditional variances', which are of clear interest as for welfare evaluations (see, e.g., Woodford (2003)).

Table 2 - Panel A collects the contribution of the four structural shocks

[18]According to Kass and Raftery (1995), a Bayes factor between 1 and 3 is "not worth more than a bare mention", between 3 and 20 suggests a "positive" evidence in favor of one of the two models, between 20 and 150 suggests a "strong" evidence against it, and larger than 150 "very strong" evidence.

(cost-push, non-policy demand, monetary policy, and trend inflation) as for the three variables of interest (output, inflation, the policy rate). The 8-step ahead decomposition assigns a dominant role of the cost-push shock as for the business cycle fluctuations of output, with a contribution over 90%. Inflation is explained by an ensemble of shocks, with the non-policy demand shock providing the largest contribution (40%), the supply shock a quite substantial one (about 30%), the policy shock playing an important role (about 20%), and the trend inflation shock being the responsible of as much as 10% of the forecast error variance decomposition. The demand shock turns out to be a key-driver for the policy rate as well, whose volatility at business cycle frequencies is also importantly determined by the policy shock and the cost-push shock. Differently, the trend inflation shock plays a marginal role here.

Trend inflation shocks are shocks to the low-frequency component of inflation. One may therefore argue that different results may be obtained when focusing on the $\infty-$step ahead decomposition. Table 2 - Panel B offers support to this intuition. First, the role of trend inflation is estimated to be large as for the volatility of inflation and the policy rate, with about 37% of the former and 25% of the latter explained by changes in the Federal Reserve's inflation target. Trend inflation shocks compete with 'supply' shocks in determining the volatility of these two variables, with the latter shocks remaining the main drivers of inflation and the short-term policy rate. The role of standard monetary policy shocks is marginal, even if they explain about 11% of the sample variance of the policy rate. The contribution of 'demand' shocks is noticeable, with a share larger than 10% as for both variables. Interestingly, trend inflation shocks do not contribute to explain the volatility of our empirical measure of the output gap, which is almost entirely explained by supply shocks. This is far from surprising, because shocks to the supply side of the economy (above all, those hitting inflation first) are typically those responsible for the inflation-output volatility trade-off in this model.

What if an econometrician failed to model trend inflation shocks? Table 2 - panels C and D reports the figures obtained by estimating a restricted version of our model that features a fixed inflation target, i.e., with 'muted' trend inflation shocks. Interestingly, evident distortions affect the identification of the drivers of the U.S. economic environment. Conditional on the analysis at business cycle frequencies, one can detect a strong over-estimation of the contribution of the demand shock as for the volatility of output. The participation of the cost-push shock to the forecast error of inflation gets doubled, while that of the policy rate turns out to be severely underestimated. Finally, the supply shock's participation as for the volatility of the policy rate is estimated to be around 50% larger. As far as our unconditional volatilities are concerned, the contribution of the supply shock to the volatilities of inflation and the federal funds rate turns out to be dramatically inflated, while the participation of demand and policy shocks gets slightly reduced. This exercise suggests that, conditional on the post-WWII U.S. sample, the omission of trend inflation may return estimated moments that are heavily distorted by model misspecification.

Our framework does not explicitly model variations in the natural level of the real interest rate, which is typically estimated to be very persistent. Therefore, the dynamics of such omitted factor could in principle bias our inflation target's estimated standard deviation upward and inflate its contribution as for the forecast error variance decomposition of inflation and the federal funds rate. To control for the effects of this model misspecification, we estimate our model by employing filtered measures of the federal funds rate that retains cyclical frequencies only. Such cyclical representation of the policy rate is obtained by subtracting Aruoba and Schorfheide's proxy of trend inflation from the federal funds rate series. We then estimate our model with such 'purged' measure of the federal funds rate along with our observables for inflation, the inflation target, and the output gap. A variance-decomposition exercise points to a reduction of the contribution of trend inflation as for the volatility of inflation and the policy rate, with figures reading 25% as for the former and 17% as for the latter (figures referring to the $\infty - step$ ahead decomposition). The remaining 'explanatory power' is assigned to supply shocks, whose contribution to the variance of inflation (federal funds rate) goes up to 59% (57%). However, the main message of our benchmark exercise, i.e., the large role played by trend inflation shocks in explaining the dynamics of the nominal side of the economy, remains qualitatively unaffected.

Is our variance decomposition analysis affected by the employment of Aruoba and Schorfheide's (2011) 'observable' for the inflation target? Table 3 contrasts the variance decomposition of our baseline scenario (already collected in Table 2, and re-proposed here for easing the reader) with that conditional on the model being estimated without any observable as for trend inflation. As for trend inflation shocks, the largest variation when not exploiting the proxy in the estimation phase regards the forecast error for inflation at business cycle frequencies (whose figures are collected in Panels A and C), which basically turns out to be twice as much as what recorder in the baseline scenario, which is, some 20%. As a matter of fact, however, the shock which gains the most in absolute terms is the supply shock, which goes up to 58% (from 28%). The identification of the drivers of the inflation rate at business cycle frequencies are importantly affected by the choice of not employing the observable for the inflation target. The demand shock's contribution to inflation volatility is some 40% when the proxy is used in the estimation, and some 20% when it is not, while that of the monetary policy shock is about 20% when the target is 'observable' vs. a much more moderate 2.53% when it is not. This latter shock's contribution to the forecast error of the policy rate turns out to be halved when dropping Aruoba and Schorfheide's (2011) proxy from the list of observables employed in the estimation of our DSGE model.

Moving to the $\infty - step$ ahead decomposition (Table 3, panels B and D), one can notice that, while being present, the differences between the scenario with vs. without an observable for trend inflation are much milder. As for the variance decomposition of output, differences are in fact negligible. Moving to inflation's, we notice in particular a larger role assigned to supply shocks when trend inflation is not 'observable' (from 44% to 56%), a substantial reduction in that of non-policy demand shocks (from 12% to 3.5%) and of policy shocks (from 6.71% to 0.45%). Finally, when considering the forecast error variance decomposition of the policy rate, we notice an increase in the participation of the supply shocks (the difference amounts to about 8%), a reduction in the contribution of demand and the policy shocks (some 5% and 8.3%, respectively), and an increase in that of the trend inflation shock of about 6%.

Our results on the $\infty - step$ ahead forecast error variance decomposition turn out to be somewhat more robust to the omission of the observable for trend inflation with respect to analysis conducted at business cycle. Moreover, the unconditional forecast error variance decomposition is of clear interest from policy purposes due to its link with microfounded welfare indicators (for a discussion, see Woodford (2003)). Finally, it allows us to draw comparisons with some relevant contributions which dealt with trend inflation shocks (e.g., Ireland (2007), Cogley, Primiceri, and Sargent (2010)). Therefore, the remainder of the paper will focus on the $\infty - step$ ahead forecast error variance decomposition.

So far, we have scrutinized a fixed sample of U.S. data, 1965:I-2005:IV. Obviously, one may wonder how stable the contribution of trend inflation is over time. In the attempt to investigate the role of trend inflation shocks further, Figure 2 reports inflation and the federal funds rate as modeled by our DSGE framework (baseline scenario) along with their counterfactual counterparts computed by setting trend inflation shocks to zero at all times. Evidently, the relative contribution of trend inflation shocks to inflation and the policy rate is far from being stable. In particular, trend inflation shocks play a great role in explaining inflation and the policy rate during the great inflation phase of the 1970s. Differently, the role of variations in trend inflation is almost negligible during the great moderation. Clearly, this evidence calls for a deepening of the role of trend inflation over time, which requires the employment of a more flexible approach as for the estimation of our DSGE model. Our choice is to undertake a rolling-window investigation, which is developed in the following Section.

4 Rolling-window approach

We then move to the investigation of the possible instabilities affecting this model's relationships by implementing a rolling-window approach. In particular, we start from the 1965:I-1979:IV window and estimate the model, then we move the first and last observation of the window by two years and repeat the estimation. We keep the size of the window fixed (at 60 observations, which is a 15-year window) to minimize the differences in the precision of our estimates due to the sample-size. Our last window covers 1991:I-2005:IV, i.e. we consider fourteen different windows, which enable us to assess fourteen different posterior densities for all the parameters of interest.

The width of our window is fairly in line with the one chosen by previous contributions when estimating small- or medium-scale DSGE models with this technique. Canova (2009) works with a window-size of 80 observations (20 years) with a small-scale model with constant trend inflation. Castelnuovo (2012a) employs 60 observations (15 years) with a small-scale framework featuring real balances in the equilibrium equations of inflation and output as well as a positive reaction of the Federal Reserve to the growth rate of nominal money. Canova and Ferroni (2012) scrutinizes a medium-scale model à la Smets and Wouters (2007) by focusing on windows of 68 quarters (17 years). A similar choice is made by Giacomini and Rossi (2010), who stick to a 70-quarter window size. Cantore, Ferroni, and Léon-Ledesma (2011) choose a window-size of 60 observations (15 years) as a benchmark, but explores alternative sizes up to 90 quarters. Our choice of a window-size of 15 years appears to be reasonable in light of the number of parameters present in the models we aim at estimating. Much smaller sizes, between 16 and 39 observations, are actually suggested by Inoue, Rossi, and Jin (2011), who develop a methodology to select the size of the window in the forecasting context when multiple models are jointly evaluated. Our purpose is clearly different, in that we aim at providing an ex-post description of the data. To achieve our goal, our sample numerosity must be high enough to allow the data to influence the posterior density via their impact on the likelihood-function.

4.1 Window-specific parameters

Figure 3 plots some selected percentiles of the posterior densities of our structural parameters against the windows considered in our estimation. We notice some instability as for the slope of the Phillips curve, which drops when the mid-1980s and 1990s are considered. Some previous literature (e.g., Primiceri (2006), Best (2011)) found the slope of the NKPC to be unstable mostly during the mid-1960s to the late 1970s. Our result lines up with the empirical evidence on the reduction of such a slope occurring in sync with the advent of the Great Moderation (see Carlstrom, Fuerst, and Paustian (2009) for a discussion). We also find instabilities in the weight of expected output in the IS curve, as well as the persistence of the cost-push shock, again when the window 1988-1999 is taken into account. The inverse of the intertemporal elasticity of substitution and the systematic policy reaction to inflation grow over time. Also the degree of interest rate smoothing and the persistence of the monetary policy shock appear to increase over time. The reaction of the output gap features ups and downs. Overall, however, the credible sets of these parameters hardly display important differences when moving from a window to another.

Figure 4 plots the evolution of the volatilities of our shocks. A number of considerations can be made. First, the volatility of supply and demand shocks display a downward (although non-monotonic) trend. Second, the volatilities of our monetary policy shocks, i.e., the standard innovation to the policy rate and shocks to trend inflation, feature a hump-shaped volatility over our windows. Both volatilities increase over the first five windows, peak at 1973-1987, then slowly decrease as the observations of 1980s and 1990s become dominant in the windows under investigation. This result supports the findings in Justiniano and Primiceri (2008) and Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) on the relevance of modeling time-conditional variances of the structural shocks identified in DSGE models.

This picture is intriguing in light of the evolution of the volatility of actual inflation and the federal funds rate, which is depicted in Figure 5. Clearly, this hump-shaped pattern is a feature of the U.S. data as for these two variables, as also documented by Canova (2009). Interestingly, the correlation between the volatility of the shocks affecting trend inflation and that of actual inflation (as measured by its sample standard deviation) is 0.64, larger than the one involving the volatility of the policy rate and trend inflation shocks, which reads 0.53. Even larger the correlations between these observables' sample standard deviations and the volatility of the standard policy rate shock, which read 0.91 and 0.80 as for inflation and the policy rate, respectively.

4.2 Window-specific variance decomposition

The presence of subsample instabilities as those documented in the previous Section naturally calls to recompute the variance decomposition of our observables by accounting for the time-dependence of our estimated parameters. Figure 6 collects the outcome of our exercise. Various considerations are in order. The relative contribution of trend inflation and cost-push shocks is clearly time-dependent. Trend inflation shocks explain about 28% of the volatility of inflation at the beginning of the sample. Then, these shocks' participation becomes larger, and hits its highest level in 1979-1993 with a share of about 62%. Subsequently, their contribution drastically declines and ends up being in line with that of the demand shock at the end of the sample, i.e., 12%. This 'inverted-U' relationship is negatively correlated with the contribution offered by the cost-push shock, which starts off at a level as high as 59%, then declines to about 19% in 1979-1993, and increases again in the last windows, with a contribution in 1991-2005 of about 63%. Differently, the contribution of demand and standard monetary policy shocks is quite stable over time, with the former being responsible for about 12% of the volatility of inflation and the latter about 8%.

The 'inverted-U' contribution by trend inflation also applies as for the volatility of the policy rate, with a participation of about 20% during the period 1965-1979, around 49% in 1977-1991, and a decline in the second part of the sample under investigation to its lowest figure - 9% - in 1991-2005. The behavior of the participation of the cost-push shock is more complex, in that it records its highest values at the beginning and at the end of the sample - 60% and 70%, respectively - and 'oscillates' in the middle of it, with the lowest value - 12% - in 1971-1989. Again, the contribution of the remaining demand shocks is fairly constant over time, with a share of about 19% attributable to the shock hitting the IS curve, and 9% to the standard monetary policy shock.

Our results suggest that i) the contribution of trend inflation shocks to the volatility of the nominal side of the economy is substantial; ii) such contribution is time-varying; iii) a description of the post-WWII U.S. economy based on the 'Volcker-appointment break' only, which would call for an analysis contrasting the great inflation phase and the great moderation period, would probably fail to capture the richness of the evolution of the participation of the trend inflation shocks over time.

5 A larger-scale DSGE model

Of course, a poor description of the processes that drive inflation and the policy rate in the U.S. economy may in principle lead to an overestimation of the role of trend inflation. As a matter of fact, virtually all central banks and a large number of researchers have drifted their attention to the richer medium-scale framework à la Smets and Wouters (2007) for some years now. This model features a variety of nominal and real frictions as well as a set of shocks that can be given a structural interpretation. We then re-propose our analysis by considering an estimated version of the Smets and Wouters' (2007) model, whose shocks can be given a structural interpretation. We refer to Smets and Wouters (2007) and to our Appendix for a full description of the model.

5.1 Data and priors

We first estimate Smets and Wouters' (2007) framework with Bayesian techniques over the sample 1965:I-2005:IV. We use the seven observables employed by Smets and Wouters (2007) (quarterly growth rates of GDP, consumption, investments, and wages, all expressed in per-capita, real terms; log of hours; GDP deflator quarterly inflation; and federal funds rate) plus the measure of trend inflation developed by Aruoba and Schorfheide (2011). As done with the small-scale model, we consider a Taylor rule in which the policymakers react to a measure of the inflation gap (as opposed to raw inflation) determined by considering an exogenous process for trend inflation as the one described by eq. (4).

The model features a deterministic growth rate driven by labor-augmenting technological progress, so that the data do not need to be detrended before estimation. Tables 3 and 4 document the priors we employed, which are the same as Smets and Wouters' (2007).

5.2 Posteriors

Our results are in line with most of the literature focusing on the estimation of DSGE models for the U.S. economy with great moderation data. In particular, we find a strong systematic policy reaction to inflation, a mild reaction to the model-consistent output gap, and a slightly stronger one to output growth. Monetary policy is conducted with a fair amount of gradualism. Our evidence points to a fairly large degree of habit formation in consumption, and lends support to the modeling of frictions in capital formation. The posterior means of the Calvo price and wage parameters are comparable with a large number of estimates obtained with macroeconomic U.S. data. Shocks to TFP, Government spending, price and wage mark-ups feature a high degree of correlation, also considering the MA(1) component of these last two shocks. Tables 3 and 4 collect some selected percentiles of our posterior densities.

5.3 Variance decomposition: Full sample analysis ...

We appeal to the variance decomposition analysis to gauge the role of trend inflation vs. other shocks conditional on the estimated medium-scale framework à la Smets and Wouters (2007). Table 5 - Panel A collects the figures computed by considering our model with trend inflation vs. an alternative that features a fixed inflation target. Interestingly, the results obtained with our small-scale model (see previous Section) are fully confirmed by the analysis with the richer Smets and Wouters' (2007) framework. Shocks to trend inflation are responsible for most of the volatility of inflation and almost half the volatility of the policy rate. Such shocks are just negligible as for the dynamics of real variables. Again, when employing trend inflation to control for the effects of the (otherwise unmodeled) low-frequency component of the federal funds rate, we record a reduction of the contribution of trend inflation shocks as for the policy rate and inflation. However, such reduction is marginal as for the policy rate (45% explained by trend inflation shocks), and more marked, but far from overturning our main message, as for inflation (47% explained by our inflation target shocks).

When omitting trend inflation shocks (Table 5 - Panel B), other shocks's contribution gets inflated. In particular, the shocks to the price and wage mark-ups turn out to be substantially magnified by the omission of a trend inflation process in the model.

As we learnt with our previous analysis, however, the contribution of trend inflation shocks is likely to be time-dependent. Therefore, we move to our rolling-window analysis and estimate the Smets and Wouters (2007) model over different windows.

5.4 ... and rolling-window investigation

Figures 7 and 8 depict the evolution of our estimated structural parameters and shocks' volatilities. While some instabilities affecting the former ones may be detected, a clear time-dependence emerges when considering the variances of our structural shocks. All shocks tend to become less volatile when moving from the 1960s and 1970s to the great moderation, with the interesting exception of the innovations affecting the wage mark-up. As before, our monetary policy shocks display a hump-shaped evolution, which makes us hint that such shocks might importantly contribute to explain the evolution of the U.S. inflation and policy rate.

Figure 9 shows the evolution of the contribution of ours shocks on the volatilities of our interest. The top panel, which focuses on inflation, is extremely similar to the one in Figure 6. The contribution of trend inflation shocks is large and time-varying, with an 'inverted-U' relationship qualitatively very similar and quantitatively even larger than the one suggested by the small scale model. 'Supply' shocks, i.e., TFP shocks and shocks to the mark-ups, play a substantial role, and tend to explain the largest share of inflation volatility towards the end of the sample. Again, we observe a fairly constant participation of 'demand' shocks (here, Government spending, risk-premium, investment-specific technology shocks) and standard monetary policy innovations, with the former explaining about 8% of the volatility of inflation, and the latter 7%.

Trend inflation shocks are estimated to be substantially relevant also as for the volatility of the policy rate. Again, the bell-shaped evolution of the contribution of such shocks is confirmed also by this medium-scale model, with a participation peaking at 49% as for the window 1979-1993. Differently with respect to the story told by the small-scale model, supply shocks play a role somewhat comparable to monetary policy shocks, with an evolution over time that appears to be complementary to that provided by standard monetary policy innovations. Interestingly, here demand shocks play a larger role than suggested by the small-scale model, with a quantitatively important, time-dependent participation that follows a U-shaped pattern hitting its peak value of 43% in the window 1989-2003.[19]

Wrapping up, estimates conducted with the Smets and Wouters (2007) framework confirm our findings, the most important ones we reiterate here: i) the contribution of trend inflation shocks to the volatility of the nominal side of the economy is substantial; ii) such contribution is time-varying; iii) a description of the post-WWII U.S. economy based on the 'Volcker-appointment break', which would call of an analysis contrasting the great inflation phase and the great moderation period, would probably represent just a part of the richness of the evolution of the participation of the trend inflation shocks over time.

6 Conclusions

This paper investigated the role of shocks to trend inflation for the post-WWII U.S. economic environment. Two new-Keynesian models of the business cycle, a small-scale AD/AS model à la Woodford (2003) and a medium-scale framework à la Smets and Wouters (2007) were modified to account for the time-varying inflation target possibly pursued by the Federal Reserve during the period 1965-2005. A mapping between the model-consistent, latent trend inflation process and the trend inflation estimate recently proposed by Aruoba and Schorfheide (2011) was imposed in the estimation. Particular attention was posed to the time-dependence of the role of the shocks to trend inflation identified with our estimated frameworks. Such time-dependence was assessed by appealing to a rolling-window approach, which enabled us to gauge the variations in the estimated parameters featured by our models.

[19] The outcome of our rolling-window exercises are robust to moderate variations of the width of our windows.

Our main findings point to a substantial contribution of trend inflation shocks in determining the volatility of variables such as inflation and the policy rate. Such contribution is found to be larger than that assigned to standard monetary policy innovations. The relative importance of trend inflation shocks in highest when observations belonging to the 1980s dominate our investigated subsamples, and less relevant (but still very relevant) during the 1970s and especially the 1990s. In contrast, we find the dynamics of the real side of the U.S. economy to be hardly explained by trend inflation shocks.

While our assumption of an exogenous trend inflation process made it possible to appreciate its role from an empirical standpoint, it is clearly unsatisfactory when turning to using these models for policy purposes. A structural interpretation for the low-frequency component of inflation is clearly needed. Interesting research pointing to the role of that learning processes of the structure of the economy by the Fed may have played in shaping the low frequencies of the U.S. inflation process have been proposed by, among others, Cogley and Sargent (2005b), Primiceri (2006), Sargent, Williams, and Zha (2006), and Carboni and Ellison (2009), and Milani (2009). Milani (2007) and Milani and Rajbhandari (2011) have shown that models that feature learning mechanisms turn out to be empirically superior than models endowed with standard rational expectation-formation as for the post-WWII U.S. macroeconomic data. Paving the 'learning avenue' is likely to be a particularly exciting research agenda for the years to come.

Another worth-exploring route relates to the distinction between policy surprises and policy 'news'. On top of policy surprises, i.e., unexpected departures from the Taylor rate, policy 'news', which are expected policy moves by the Federal Reserve due to communications, announcements, and the like, are potentially powerful shocks in terms of impact on the economic environment, as suggested by some recent empirical investigations (Milani and Treadwell (2011)). Clearly, a distinction between news to the federal funds rate conditional on a fixed-inflation target vs. news to the policy rate due to an expected variation in such target can be made.

Finally, our results are obtained via Bayesian estimations conditional on a time-domain approach. In two recent papers, Tkachenko and Qu (2011a,b) study parameter identification, estimation, and inference in medium-scale

DSGE models from a frequency-domain perspective. In a recent contribution, Sala (2011) conducts a similar effort. Such approach naturally leads them to focus on the business cycle frequencies of interest for DSGE model-builders. We see the investigation of the role of trend inflation shocks undertaken with the lenses offered by a frequency-domain approach as an intriguing effort for future research.

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Appendix

A1 - Bayesian estimation

To perform our Bayesian estimations we employed `DYNARE`, a set of algorithms developed by Michel Juillard and collaborators (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011)). `DYNARE` is freely available at the following URL: http://www.dynare.org/.

The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first $t \leq t_0 = 20,000$ draws. To do so, we computed the posterior mode by the 'csminwel' algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix C_0 of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the 'Adaptive Metropolis' (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient than the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to 'tune' the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix C_t of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, see Haario, Saksman, and Tamminen (2001).

As for the small-scale three-equation DSGE model, we simulated two chains of 400,000 draws each, and discarded the first 90% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations. When turning to the Smets and Wouters' (2007) model, we employed 100,000 draws of the Metropolis-Hastings to simulate the posterior density of such framework.

A2 - The Smets-Wouters (2007) model

The Smets and Wouters (2007) model is a Dynamic Stochastic General Equilibrium framework extremely popular in academic and institutional circles. The model features a number of shocks and frictions, which offer a quite rich representation of the economic environment and allow for a satisfactory in-sample fit of a set of macroeconomic data (Del Negro, Schorfheide, Smets, and Wouters (2007)). Moreover, Smets and Wouters (2007) show that this model is quite competitive when contrasted with Bayesian-VARs as for forecasting exercises, in particular for the elaboration of medium-term predictions.

The Smets and Wouters (2007) model features sticky nominal price and wage settings that allow for backward-looking inflation indexation; habit formation in consumption; investment adjustment costs; variable capital utilization and fixed costs in production. The stochastic dynamics is driven by seven structural shocks, namely a total factor productivity shock, two shocks affecting the intertemporal margin (risk premium shocks and investment-specific technology shocks), two shocks affecting the intratemporal margin (wage and price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks).

In a nutshell, the model features the following main ingredients. Households maximize a nonseparable utility function in consumption and labor over an infinite life horizon. Consumption appears in the utility function in quasi-difference form with respect to a time-varying external habit variable. Labor is differentiated by a union, so there is some monopoly power over wages, which results in explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. The utilization of the capital stock can be adjusted at increasing cost. Firms produce differentiated goods, decide on labor and capital inputs, and set prices conditional on the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that are not reoptimized are partially indexed to past inflation rates. Prices are therefore set in function of current and expected marginal costs, but are also determined by the past inflation rate. The marginal costs depend on wages and the rental rate of capital. Similarly, wages depend on past and expected future wages and inflation. The model features, in both goods and labor markets, an aggregator that allows for a time-varying demand elasticity depending on the relative price as in Kimball (1995). This is important because the introduction of real rigidity allows us to estimate a more reasonable degree of price and wage stickiness.

The log-linearized version of the DSGE model around its steady-state growth path reads as follows:

$$
\begin{aligned}
y_t &= c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g && (7)\\
c_t &= c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \varepsilon_t^b) && (8)\\
i_t &= i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i && (9)\\
q_t &= q_1 E_t q_t + 1 + (1 - q_1) E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \varepsilon_t^b) && (10)\\
y_t &= \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a) && (11)\\
k_t^s &= k_{t-1} + z_t && (12)\\
z_t &= z_1 r_t^k && (13)\\
k_t &= k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i && (14)\\
\mu_t^p &= \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t && (15)\\
\pi_t &= \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p && (16)\\
r_t^k &= -(k_t - l_t) + w_t && (17)\\
\mu_t^w &= w_t - (\sigma_l l_t + (1 - \lambda/\gamma)^{-1} (c_t - \lambda/\gamma c_{t-1})) && (18)\\
w_t &= w_1 w_{t-1} + w_2 (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w && (19)\\
r_t &= \rho r_{t-1} + (1 - \rho)(r_\pi + r_Y (y_t - y_t^p)) + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^R && (20)\\
\varepsilon_t^x &= \rho_x \varepsilon_{t-1}^x + \nu_t^x, x = (b, i, a, R) && (21)\\
\varepsilon_t^g &= \rho_g \varepsilon_{t-1}^g + \nu_t^g + \rho_{ga} \nu_t^a && (22)\\
\varepsilon_t^z &= \rho_x \varepsilon_{t-1}^z + \nu_t^z - \chi_z \nu_{t-1}^z, z = (p, w) && (23)\\
\nu_t^j &\sim N(0, \sigma_j^2) && (24)
\end{aligned}
$$

where:

$$c_y = 1 - g_y - i_y \tag{25}$$

and g_y and i_y are the steady-state exogenous spending-output ratio and investment-output ratio, with:

$$i_y = (\gamma - 1 + \delta) k_y \tag{26}$$

where γ is the steady-state growth rate, δ is the depreciation rate of capital, k_y is the steady-state capital-output ratio; $z_y = R_*^y k_y$ is the steady-state rental rate of capital. Notice that eq. (22), the one of the stochastic process of the government spending, allows for the productivity shock to affect it.

This is so because exogenous spending, in this model, includes net exports, which may be affected by domestic productivity development.

As for the consumption Euler equation (8):

$$c_1 = \frac{\lambda}{\gamma}\left(1+\frac{\lambda}{\gamma}\right) \tag{27}$$

$$c_2 = \frac{(\sigma_c - 1)\frac{W_*^h L_*}{C_*}}{\sigma_c(1+\frac{\lambda}{\gamma})} \tag{28}$$

$$c_3 = \frac{1-\frac{\lambda}{\gamma}}{\left(1+\frac{\lambda}{\gamma}\right)\sigma_c} \tag{29}$$

Current consumption is a function of past and expected future consumption, of expected growth in hours worked, of the ex ante real interest rate, and of a disturbance term ε_t^b. Under the assumption of no habits ($\lambda = 0$) and that of log-utility in consumption ($\sigma_c = 1$), $c_1 = c_2 = 0$, then the standard purely forward looking consumption equation is obtained. The disturbance term ε_t^b represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive shock to this wedge increases the required return on assets held by the households. At the same time, it increases the cost of capital and it decreases the value of capital and investment (see below). This is basically a shock very similar to a net-worth shock. This disturbance is assumed to follow a standard AR(1) process.

The dynamics of investment is captured by the investment Euler equation (9), where:

$$i_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}} \tag{30}$$

$$i_2 = \frac{1}{1+\beta\gamma^{1-\sigma_c}\gamma^2\varphi} \tag{31}$$

where φ is the steady-state elasticity of the capital adjustment cost function, and β is the discount factor applied by households. Notice that capital adjustment costs are a function of the change in investment, rather than its level. This choice is made to introduce additional dynamics in the investment equation, which is useful to capture the hump-shaped response of investment to various shocks. In this equation, the stochastic disturbance ε_t^i represents a shock to the investment-specific technology process, and is assumed to follow a standard first-order autoregressive process.

The value-of-capital arbitrage equation (10) suggests that the current value of the capital stock q_t depends positively on its expected future value (with weight $q_1 = \beta\gamma^{-\sigma_c}(1-\delta)$), as well as the expected real rental rate on capital $E_t r_{t+1}^k$ and on the ex ante real interest rate and the risk premium disturbance.

Eq. (11) is the first one of the supply side block. It describes the aggregate production function, which maps output to capital (k_t^s) and labor services (l_t). The parameter α captures the share of capital in production, and the parameter ϕ_p is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

Eq. (12) suggest that the newly installed capital becomes effective with a one-period delay, hence current capital services in production are a function of capital installed in the previous period k_t and the degree of capital utilization z_t. As stressed by eq. (13), the degree of capital utilization is a positive function of the rental rate of capital, $z_t = z_1 r_t^k$, where $z_1 = (1-\psi)/\psi$ and ψ is a positive function of the elasticity of the capital utilization adjustment cost function normalized to belong to the [0,1] domain.

Eq. (14) describes the accumulation of installed capital k_t, featuring the convolutions:

$$k_1 = (1-\delta)/\gamma \tag{32}$$

$$k_2 = \left[1-\left(1-\frac{\delta}{\gamma}\right)\right]\left(1+\beta\gamma^{1-\sigma_c}\right)\gamma^2\varphi \tag{33}$$

Installed capital is a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance ε_t^i, which follows an autoregressive, stationary process.

Eq. (15) relates to the monopolistic competitive goods market. Cost minimization by firms implies that the price mark-up μ_t^p, defined as the difference between the average price and the nominal marginal cost or the negative of the real marginal cost, is equal to the difference between the marginal product of labor and the real wage w_t, with the marginal product of labor being itself a positive function of the capital-labor ratio and total factor productivity.

Profit maximization by price-setting firms gives rise to the New-Keynesian Phillips curve, i.e., eq. (16), with the convolutions being:

$$\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}, \tag{34}$$

$$\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}, \tag{35}$$

$$\pi_3 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{\xi_p\left[(\phi_p-1)\varepsilon_p+1\right]}. \tag{36}$$

Notice that, in maximizing their profits, firm have to face price stickiness à la Calvo (1983). Firms that cannot reoptimize in a given period index their prices to past inflation as in Smets and Wouters (2003). In equilibrium, inflation π_t depends positively on past and expected future inflation, negatively on the current price mark-up, and positively on a price mark-up disturbance ε_t^p. The price mark-up disturbance is assumed to follow an ARMA(1,1) process. The inclusion of the MA term is to grab high-frequency fluctuations in inflation. When the degree of price indexation $\iota_p = 0$, $\pi_1 = 0$ and eq. (16) collapses to the purely forward-looking, standard NKPC. The assumption that all prices are indexed to either lagged inflation or trend inflation ensures the verticality of the Phillips curve in the long run. The speed of adjustment to the desired mark-up depends, among others, on the degree of price stickiness ξ_p, the curvature of the Kimball goods market aggregator ε_p, and the steady-state mark up, which in equilibrium is itself related to the share of fixed costs in production $(\phi_p - 1)$ via a zero-profit condition. In particular, when all prices are flexible ($\xi_p = 0$) and the price mark-up shock is zero at all times, eq. (16) reduces to the familiar condition that the price mark-up is constant, or equivalently that there are no fluctuations in the wedge between the marginal product of labor and the real wage. Cost minimization by firms also implies that the rental rate of capital is negatively related to the capital-labor ratio and positively to the real wage (both with unitary elasticity) (see eq. (17)).

Similarly, in the monopolistically competitive labor market, the wage mark-up will be equal to the difference between the real wage and the marginal rate of substitution between working and consuming, an equivalence captured by eq. (18), where σ is the elasticity of labor supply with respect to the real wage and λ is the habit parameter in consumption. Eq. (19) shows that real wages adjust only gradually to the desired wage mark-up due to nominal wage stickiness and partial indexation, the convolutions related to this equation being:

$$w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}} \tag{37}$$

$$w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}} \tag{38}$$

$$w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}} \tag{39}$$

$$w_4 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}} \frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_w)(1-\xi_w)}{\xi_w\left[(\phi_w-1)\varepsilon_w+1\right]} \tag{40}$$

Notice that if wages are perfectly flexible ($\xi_w = 0$), the real wage is a constant mark-up over the marginal rate os substitution between consumption and leisure. When wage indexation is zero ($\iota_w = 0$), real wages do not depend on lagged inflation. Notice that, symmetrically with respect to the pricing scheme analyzed earlier, also the wage-mark up disturbance follows an ARMA(1,1) process.

The model is closed by eq. (20), which is a flexible Taylor rule postulating a systematic reaction by policymakers to current values of inflation, the output gap, and output growth. In particular, one of the objects policymakers react to is the output gap, defined as a difference between actual and potential output (in logs). Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks. Then, policymakers engineer movements in the short-run policy rate r_t, movements which happen gradually given the presence of interest rate smoothing ρ. Stochastic departures from the Taylor rate, i.e. the rate that would realize in absence of any policy rate shocks, are triggered by a stochastic AR(1) process.

Finally, eqs. (21)-(24) define the stochastic processes of the model, which features, as already pointed out, seven shocks (total factor productivity, investment specific technology, risk premium, exogenous spending, price mark-up, wage mark-up, and monetary policy).

Notice that the model features a deterministic growth rate driven by labor-augmenting technological progress, so that the data do not need to be detrended before estimation.

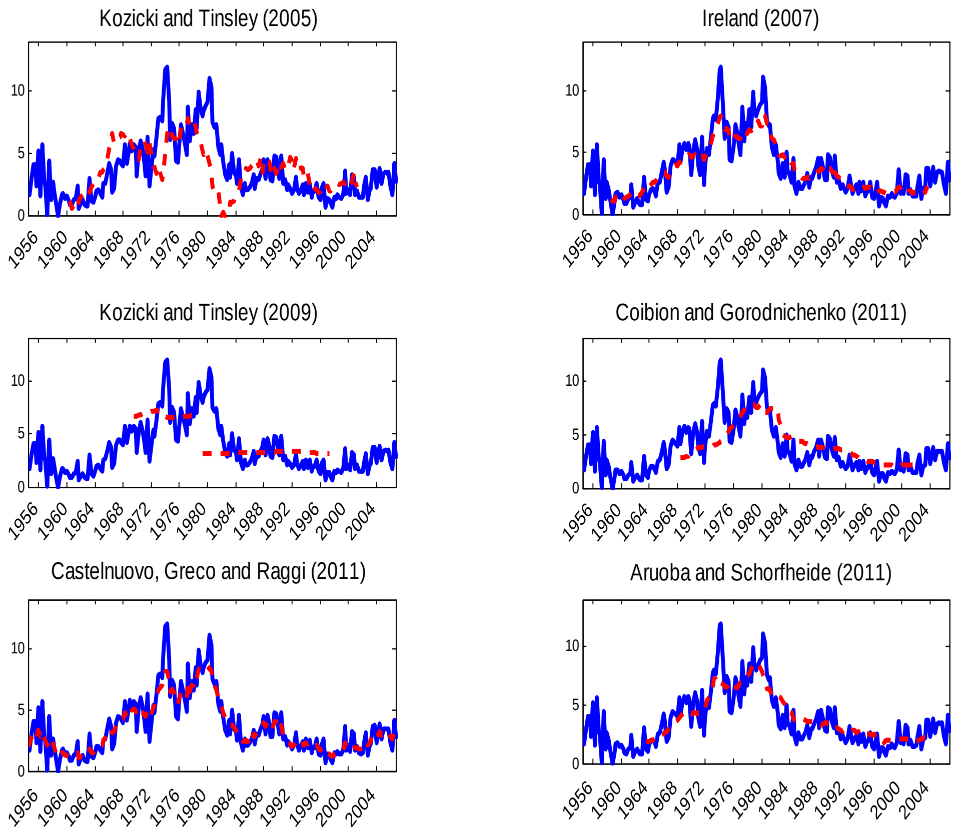

Figure 1: **Alternative estimates of trend inflation in the literature. Trend inflation estimates: A comparison.** Solid blue lines: GDP deflator quarterly inflation; red dashed lines: Trend inflation estimates, different contributions.Sources of other contributions' estimates reported in the text.

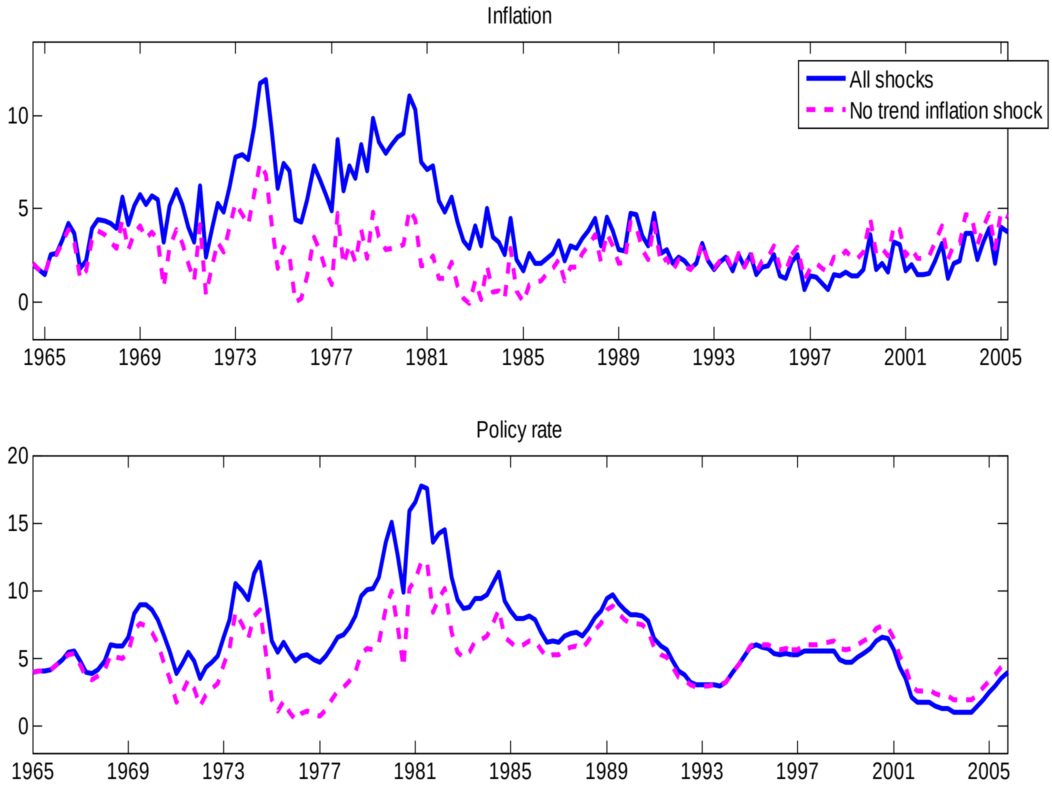

Figure 2: **Variance decomposition - role of trend inflation shocks**. Smoothed series, model calibrated with posterior mean.

Param.	*Interpretation*	*Priors* *Baseline set*	*Post.Means* [5h,95th] (1)	*Post.Means* [5h,95th] (2)	*Post.Means* [5h,95th] (3)	*Priors* *Alternative set* (4)	*Post.Means* [5h,95th] (4)
β	Discount factor	*Calibr.*	0.99 [–]	0.99 [–]	0.99 [–]	*Calibr.*	0.99 [–]
κ	NKPC, slope	$\mathcal{N}(0.1, 0.015)$	0.11 [0.09,0.13]	0.09 [0.07,0.12]	0.11 [0.09,0.13]	$\mathcal{N}(0.1, 0.015)$	0.01 [0.01,0.01]
α	Price indexation	$\mathcal{B}(0.5, 0.2)$	0.09 [0.02,0.15]	0.08 [0.01,0.15]	0.04 [0.01,0.08]	$\boldsymbol{B(0.75, 0.15)}$	0.69 [0.58,0.82]
γ	IS, forw. look. degree	$\mathcal{B}(0.5, 0.2)$	0.71 [0.66,0.76]	0.78 [0.69,0.86]	0.70 [0.64,0.76]	$\mathcal{B}(0.5, 0.2)$	0.46 [0.39,0.53]
σ	Inverse of the IES	$\mathcal{N}(3, 1)$	5.45 [4.44,6.51]	5.10 [3.91,6.30]	5.49 [4.30,6.70]	$\mathcal{N}(3, 1)$	5.46 [4.18,6.64]
ϕ_π	T. Rule, inflation	$\mathcal{N}(1.5, 0.3)$	2.22 [1.95,2.51]	2.22 [1.90,2.55]	2.16 [1.78,2.55]	$\mathcal{N}(1.5, 0.3)$	1.80 [1.43,2.15]
ϕ_x	T. Rule, output gap	$\mathcal{G}(0.3, 0.2)$	0.03 [0.01,0.05]	0.20 [0.10,0.30]	0.04 [0.01,0.07]	$\mathcal{G}(0.3, 0.2)$	0.28 [0.18,0.40]
ϕ_R	T. Rule, inertia	$\mathcal{B}(0.5, 0.285)$	0.73 [0.68,0.78]	0.75 [0.70,0.81]	0.73 [0.66,0.80]	$\mathcal{B}(0.5, 0.285)$	0.83 [0.79,0.88]
ρ_π	AR coeff. cost-push shock	$\mathcal{B}(0.75, 0.15)$	0.98 [0.97,0.99]	0.98 [0.96,0.99]	0.98 [0.97,0.99]	$\boldsymbol{B(0.25, 0.15)}$	0.06 [0.01,0.11]
ρ_x	AR coeff. demand shock	$\mathcal{B}(0.5, 0.2)$	0.70 [0.60,0.80]	0.76 [0.68,0.85]	0.68 [0.58,0.78]	$\mathcal{B}(0.5, 0.2)$	0.39 [0.24,0.57]
ρ_R	AR coeff. mon. pol. shock	$\mathcal{B}(0.5, 0.2)$	0.39 [0.25,0.49]	0.21 [0.07,0.33]	0.32 [0.17,0.46]	$\mathcal{B}(0.5, 0.2)$	0.25 [0.12,0.40]
ρ_*	AR coeff. trend infl. shock	*Calibr.*	0.995 [–]	0.995 [–]	0.995 [–]	*Calibr.*	0.995 [–]
σ_π	Std. dev. cost-push shock	$\mathcal{IG}(0.2, 0.1)$	0.14 [0.12,0.16]	0.09 [0.06,0.12]	0.13 [0.11,0.16]	$\mathcal{IG}(0.2, 0.1)$	0.17 [0.16,0.19]
σ_x	Std. dev. demand shock	$\mathcal{IG}(0.2, 0.1)$	0.16 [0.11,0.22]	0.16 [0.11,0.22]	0.15 [0.11,0.20]	$\mathcal{IG}(0.2, 0.1)$	0.36 [0.25,0.44]
σ_R	Std. dev. mon. pol. shock	$\mathcal{IG}(0.2, 0.1)$	0.29 [0.27,0.31]	0.25 [0.22,0.28]	0.28 [0.24,0.31]	$\mathcal{IG}(0.2, 0.1)$	0.25 [0.22,0.27]
σ_*	Std. dev. trend infl. shock	$\mathcal{IG}(0.2, 0.1)$	0.05 [0.04,0.06]	0.11 [0.06,0.15]	0.05 [0.04,0.06]	$\mathcal{IG}(0.2, 0.1)$	0.05 [0.04,0.06]
$log(ML)$			-13.05				-20.68

Table 1: **Bayesian estimates of the small-scale DSGE model.** 1965:I-2005:IV U.S. data. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Scenarios. (1): Baseline; (2): Estimation with trend inflation as an unobservable factor (no proxy in the measurement equation); (3): Estimation with trend inflation proxied by a one-sided Band-Pass filter applied to GDP deflator inflation; (4): Estimation with a set of alternative priors (higher prior mean on price indexation, lower prior mean on autoregressive parameter of the cost-push shock. Log-Marginal Likelihoods reported when comparable to the Baseline scenario. Details on the estimation procedure provided in the text.

		$Var/shocks.$	v^{π}	v^{y}	v^{R}	v^{*}
		Panel A				
Trend	$8 - step\ ahead$	x	92.11	5.77	2.10	0.02
inflation		π	28.21	39.53	21.74	10.52
shocks		R	20.54	42.46	32.85	4.15
		Panel B				
	$\infty - step\ ahead$	x	99.30	0.51	0.19	0.00
		π	44.12	12.19	6.71	36.99
		R	49.05	14.77	11.25	24.94
		Panel C				
No trend	$8 - step\ ahead$	x	87.29	10.29	2.42	
inflation		π	59.09	34.88	6.03	–
shocks		R	30.93	45.23	23.84	
		Panel D				
	$\infty - step\ ahead$	x	98.60	1.14	0.27	
		π	91.42	7.33	1.25	–
		R	83.51	11.36	5.13	

Table 2: **Variance decomposition implied by the small-scale DSGE model.** 1965:I-2005:IV U.S. data. Figures conditional on the posterior mode values of the model. Panel A and B: Scenarios with trend inflation shocks; Panel C and D: Scenarios without trend inflation shocks. Details on the estimation procedure provided in the text.

		$Var/shocks.$	v^{π}	v^{y}	v^{R}	v^{*}
		Panel A				
Trend inflation as 'observable'	$8-step\ ahead$	x	92.11	5.77	2.10	0.02
		π	28.21	39.53	21.74	10.52
		R	20.54	42.46	32.85	4.15
		Panel B				
	$\infty-step\ ahead$	x	99.30	0.51	0.19	0.00
		π	44.12	12.19	6.71	36.99
		R	49.05	14.77	11.25	24.94
		Panel C				
Trend inflation as latent factor	$8-step\ ahead$	x	87.57	10.62	1.63	0.18
		π	58.50	19.78	2.53	19.19
		R	34.41	42.96	14.76	7.87
		Panel D				
	$\infty-step\ ahead$	x	98.02	1.66	0.25	0.07
		π	56.31	3.53	0.45	39.71
		R	57.32	9.50	2.92	30.26

Table 3: **Variance decomposition implied by the small-scale DSGE model.** 1965:I-2005:IV U.S. data. Figures conditional on the posterior mode values of the model. Panel A and B: Scenarios with an observable for trend inflation; Panel C and D: Scenarios without an observable for trend inflation. Details on the estimation procedure provided in the text.

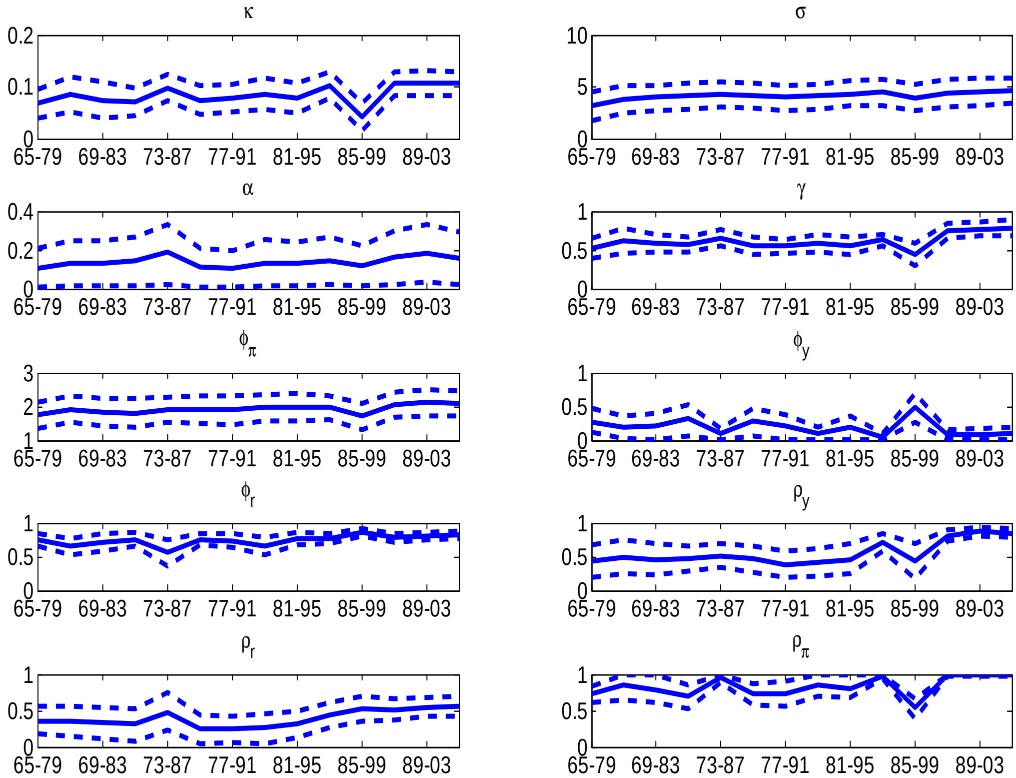

Figure 3: **Evolution of the structural parameters of our small-scale DSGE model**. Definitions of the structural parameters given in Table 1. Solid line: Posterior median. Dotted lines: 5th and 95th posterior percentiles. Evolution of the parameters constructed by employing fourteen rolling windows of 15-year constant length. Windows: [1965:I-1979:IV, 1967:I-1981:IV, ..., 1991:I-2005:IV].

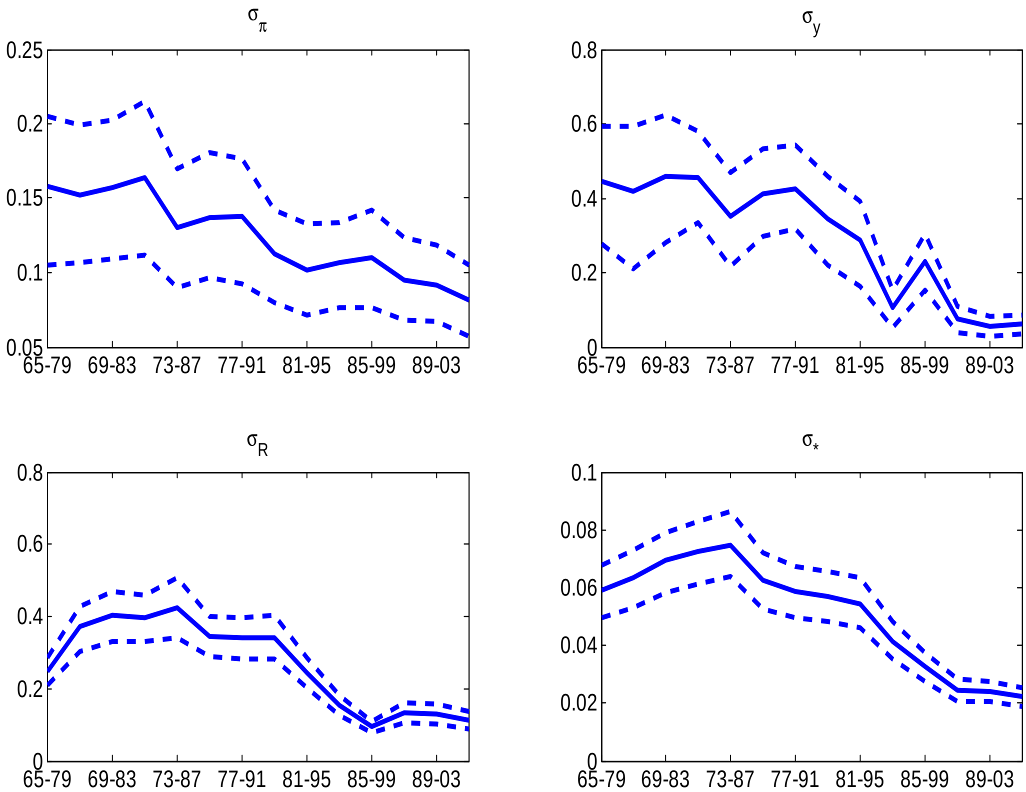

Figure 4: **Evolution of the shocks' standard deviations of our small-scale DSGE model**. Definitions of the shocks' standard deviations given in Table 1. Solid line: Posterior median. Dotted lines: 5th and 95th posterior percentiles. Evolution of the shocks' standard deviations constructed by employing fourteen rolling windows of 15-year constant length. Windows: [1965:I-1979:IV, 1967:I-1981:IV, ..., 1991:I-2005:IV].

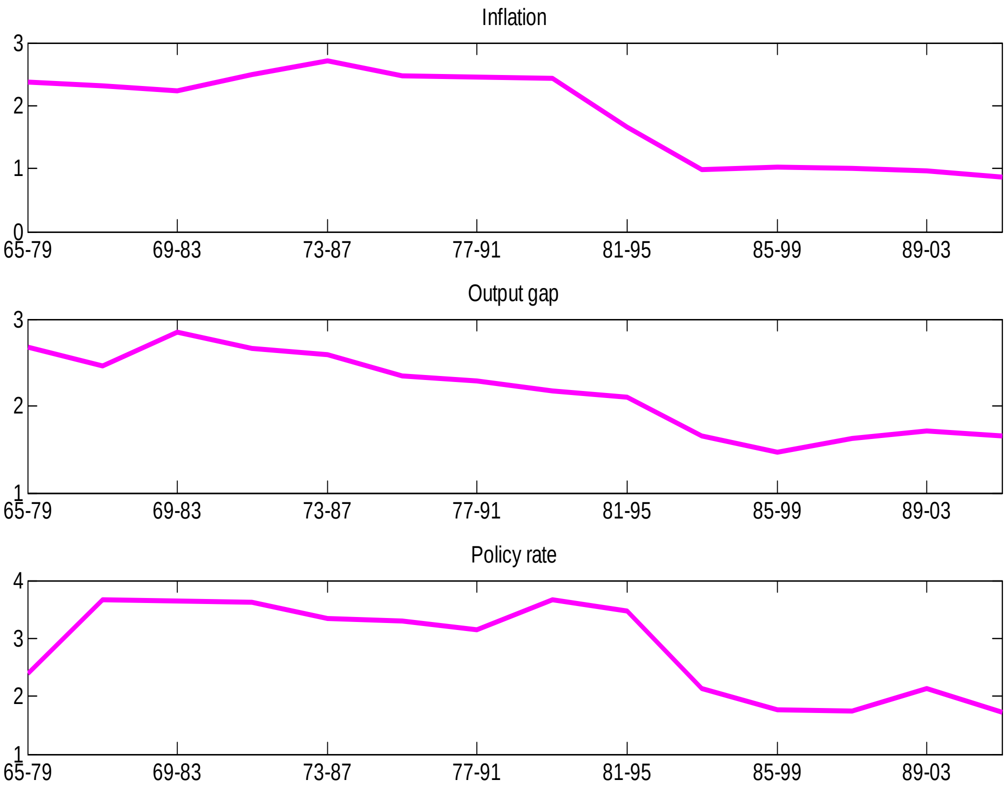

Figure 5: **Evolution of the standard deviations of our observables.** Sample moments computed by considering quarterly rates of inflation and the policy indicator. Evolution of the moments constructed by employing fourteen rolling windows of 15-year constant length. Windows: [1965:I-1979:IV, 1967:I-1981:IV, ..., 1991:I-2005:IV].

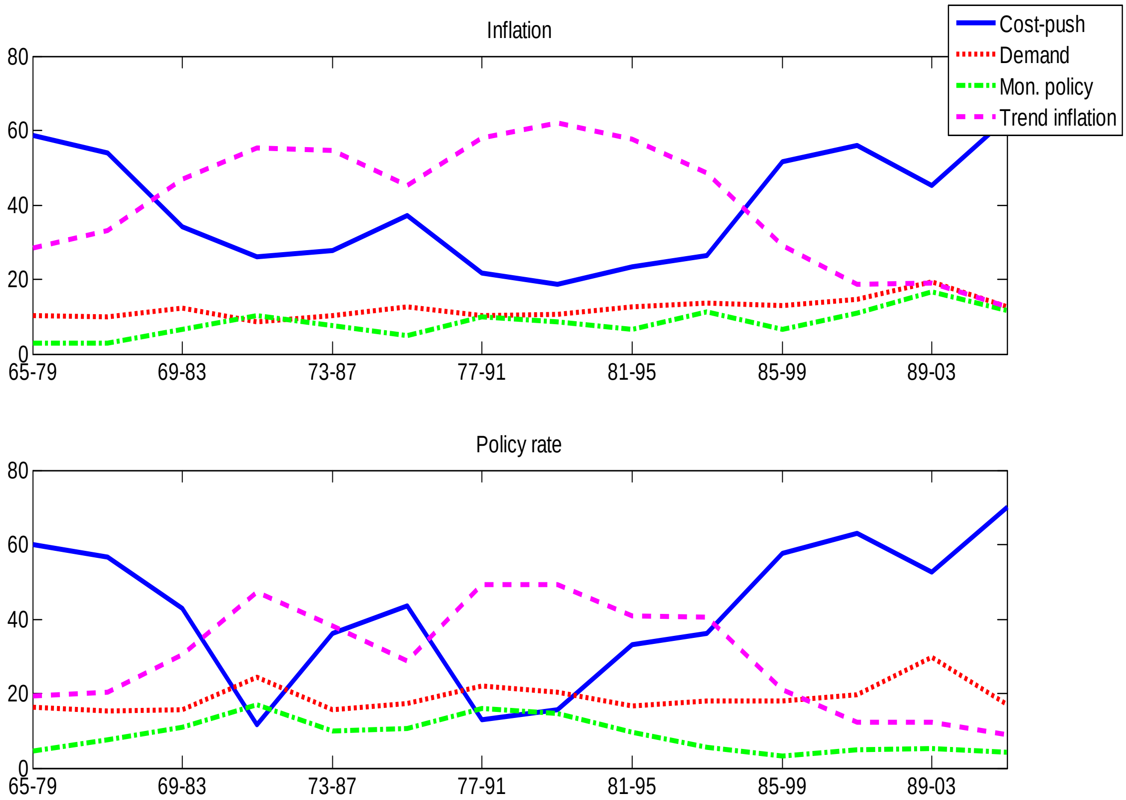

Figure 6: **Evolution of the variance decomposition implied by the estimated small-scale DSGE model.** Window-specific variance decomposition computed by calibrating the small-scale model with its posterior mode values.

$Param.$	$Interpretation$	$Priors$	$Posterior\ Means$ $_{[5h,95th]}$
φ	Capital adj. cost elasticity	$Normal(4, 1.5)$	5.72 [4.10,7.24]
σ_c	Risk aversion	$Normal(1.5, 0.375)$	1.23 [1.07,1.37]
h	Habit formation	$Beta(0.7, 0.1)$	0.73 [0.66,0.80]
ξ_w	Wage stickiness	$Beta(0.5, 0.1)$	0.65 [0.54,0.78]
σ_l	Elast. lab. supply	$Normal(2, 0.75)$	0.85 [0.05,1.65]
ξ_p	Price stickiness	$Beta(0.5, 0.1)$	0.67 [0.61,0.74]
ι_w	Wage indexation	$Beta(0.5, 0.15)$	0.58 [0.37,0.78]
ι_p	Price indexation	$Beta(0.5, 0.15)$	0.16 [0.07,0.27]
ψ	Capacity utiliz. elast.	$Beta(0.5, 0.15)$	0.57 [0.40,0.75]
$\Phi - 1$	Fixed c. in prod. (share)	$Normal(0.25, 0.125)$	0.48 [0.37,0.60]
r_π	T. Rule, inflation	$Normal(1.5, 0.25)$	2.09 [1.79,2.41]
ρ	T. Rule, inertia	$Beta(0.75, 0.10)$	0.81 [0.77,0.85]
r_y	T. Rule, output gap	$Normal(0.125, 0.05)$	0.00 [−0.01,0.02]
$r_{\Delta y}$	T. Rule, output growth	$Normal(0.125, 0.05)$	0.20 [0.15,0.24]
$\overline{\pi}$	St. state inflation rate	$Gamma(0.625, 0.10)$	0.61 [0.51,0.70]
$100(\beta^{-1} - 1)$	St. state interest rate	$Gamma(0.25, 0.10)$	0.25 [0.09,0.40]
$\bar{l}$	St. state hours worked	$Normal(0, 2)$	0.18 [−2.37,2.60]

Table 4: **Bayesian estimates of the Smets and Wouters' (2007) DSGE model - Structural Parameters.** 1984:I-2008:II U.S. data. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text.

Param.	*Interpretation*	*Priors*	*Posterior Means* [$5h$,$95th$]
σ_a	TFP shock, st.dev.	$InvGamma(0.1, 2)$	0.50 [0.46,0.55]
σ_b	Risk-premium shock, st.dev.	$InvGamma(0.1, 2)$	0.24 [0.20,0.28]
σ_g	Gov. spending shock, st.dev.	$InvGamma(0.1, 2)$	0.55 [0.50,0.52]
σ_I	Invest.-specific tech. shock, st.dev.	$InvGamma(0.1, 2)$	0.44 [0.36,0.52]
σ_r	Mon. policy shock, st.dev.	$InvGamma(0.1, 2)$	0.24 [0.22,0.27]
σ_p	Price mark-up shock, st.dev.	$InvGamma(0.1, 2)$	0.14 [0.12,0.16]
σ_w	Wage mark-up shock, st.dev.	$InvGamma(0.1, 2)$	0.27 [0.23,0.31]
σ_*	Trend inflation shock, st. dev.	$InvGamma(0.1, 2)$	0.05 [0.04,0.06]
ρ_a	TFP shock, AR(1) coeff.	$Beta(0.5, 0.2)$	0.96 [0.95,0.98]
ρ_b	Risk-premium shock, AR(1) coeff.	$Beta(0.5, 0.2)$	0.25 [0.12,0.38]
ρ_g	Gov. sp. shock, AR(1) coeff.	$Beta(0.5, 0.2)$	0.95 [0.94,0.97]
ρ_I	Invest.-spec. tech. shock, AR(1) coeff.	$Beta(0.5, 0.2)$	0.69 [0.58,0.78]
ρ_r	Mon. pol. shock, AR(1) coeff.	$Beta(0.5, 0.2)$	0.17 [0.06,0.28]
ρ_p	Price mark-up shock., AR(1) coeff.	$Beta(0.5, 0.2)$	0.92 [0.87,0.98]
ρ_w	Wage mark-up shock, AR(1) coeff.	$Beta(0.5, 0.2)$	0.98 [0.97,0.99]
μ_p	Price mark-up shock, MA(1) coeff.	$Beta(0.5, 0.2)$	0.73 [0.61,0.86]
μ_w	Wage mark-up shock, MA(1) coeff.	$Beta(0.5, 0.2)$	0.85 [0.75,0.95]
ρ_{ga}	Gov.spending-TFP shocks, correlation	$Beta(0.5, 0.2)$	0.51 [0.37,0.65]

Table 5: **Bayesian estimates of the Smets and Wouters' (2007) DSGE model - Shock processes.** 1984:I-2008:II U.S. data. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text.

	Var/shocks.	v^a	v^b	v^g	v^I	v^r	v^p	v^w	v^*
					Panel A				
Trend	HRS	1.55	1.86	5.25	7.51	2.51	8.87	72.35	*0.10*
inflation	R	5.66	7.00	2.30	16.00	12.64	3.84	3.71	*48.85*
shocks	π	2.48	1.19	0.70	3.97	6.36	16.87	11.16	*57.27*
	γ_{GDP}	14.28	18.75	26.98	20.76	6.40	6.04	6.64	*0.15*
	γ_{CON}	6.19	56.58	0.45	0.83	11.59	5.21	18.89	*0.28*
	γ_{INV}	4.89	3.48	0.66	78.92	3.48	5.83	2.66	*0.10*
	γ_{WAG}	6.64	1.45	0.05	1.98	2.02	26.45	61.33	*0.08*
					Panel B				
No trend	HRS	2.02	2.13	6.68	11.43	2.74	8.39	66.61	
inflation	R	8.60	6.34	3.94	26.78	12.54	6.98	34.82	
shocks	π	4.15	0.52	1.03	4.70	3.66	29.02	56.92	
	γ_{GDP}	16.07	18.40	26.63	22.12	5.35	5.45	5.98	–
	γ_{CON}	6.89	58.32	1.26	1.62	10.66	5.29	15.96	
	γ_{INV}	5.80	2.88	1.15	80.85	2.48	4.63	2.21	
	γ_{WAG}	5.80	0.39	0.08	2.20	0.81	27.08	63.64	

Table 6: **Variance decomposition implied by the Smets and Wouters' (2007) DSGE model.** 1965:I-2005:IV U.S. data. Figures conditional on the posterior mode values of the model. Details on the estimation procedure provided in the text.

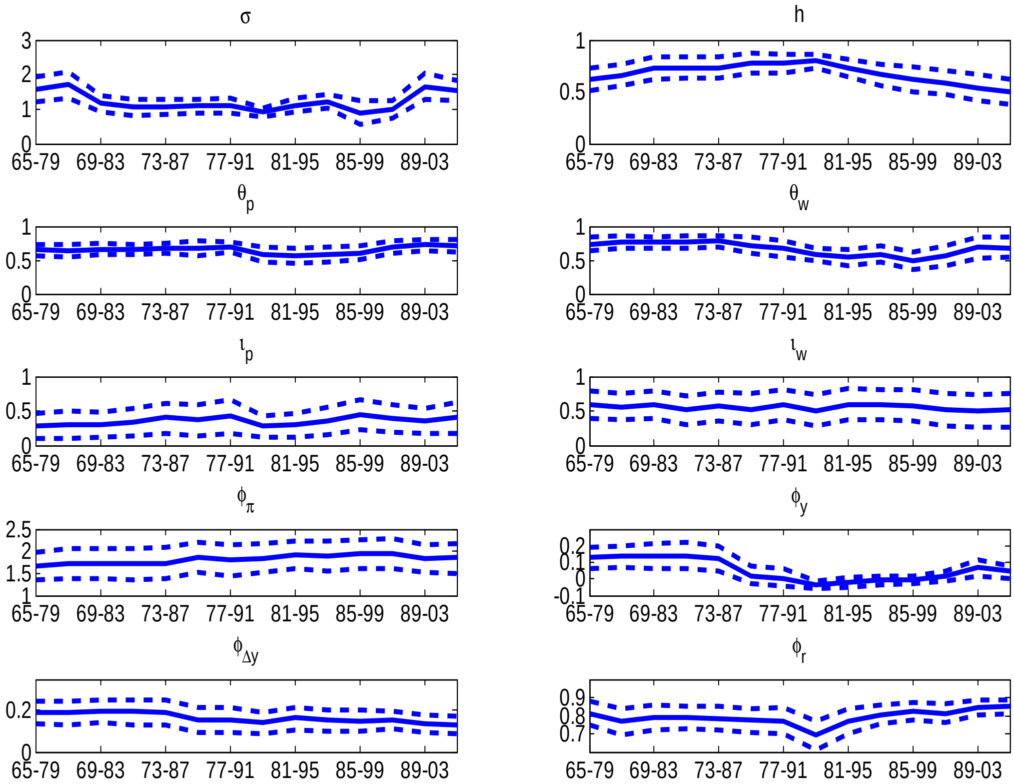

Figure 7: **Evolution of the structural parameters of the Smets and Wouters' (2007) DSGE model**. Definitions of the structural parameters given in Table 1. Solid line: Posterior median. Dotted lines: 5th and 95th posterior percentiles. Evolution of the parameters constructed by employing fourteen rolling windows of 15-year constant length. Windows: [1965:I-1979:IV, 1967:I-1981:IV, ..., 1991:I-2005:IV].

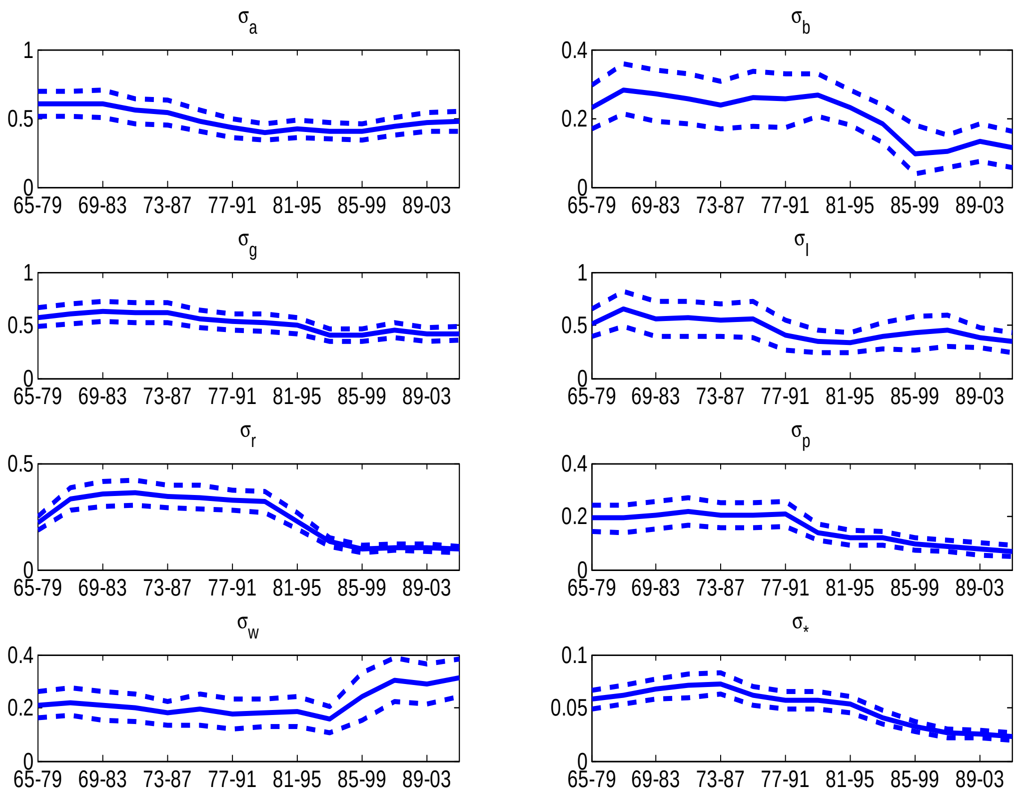

Figure 8: **Evolution of the shocks' standard deviations of the Smets and Wouters' (2007) DSGE model**. Definitions of the shocks' standard deviations given in Table 1. Solid line: Posterior median. Dotted lines: 5th and 95th posterior percentiles. Evolution of the shocks' standard deviations constructed by employing fourteen rolling windows of 15-year constant length. Windows: [1965:I-1979:IV, 1967:I-1981:IV, ..., 1991:I-2005:IV].

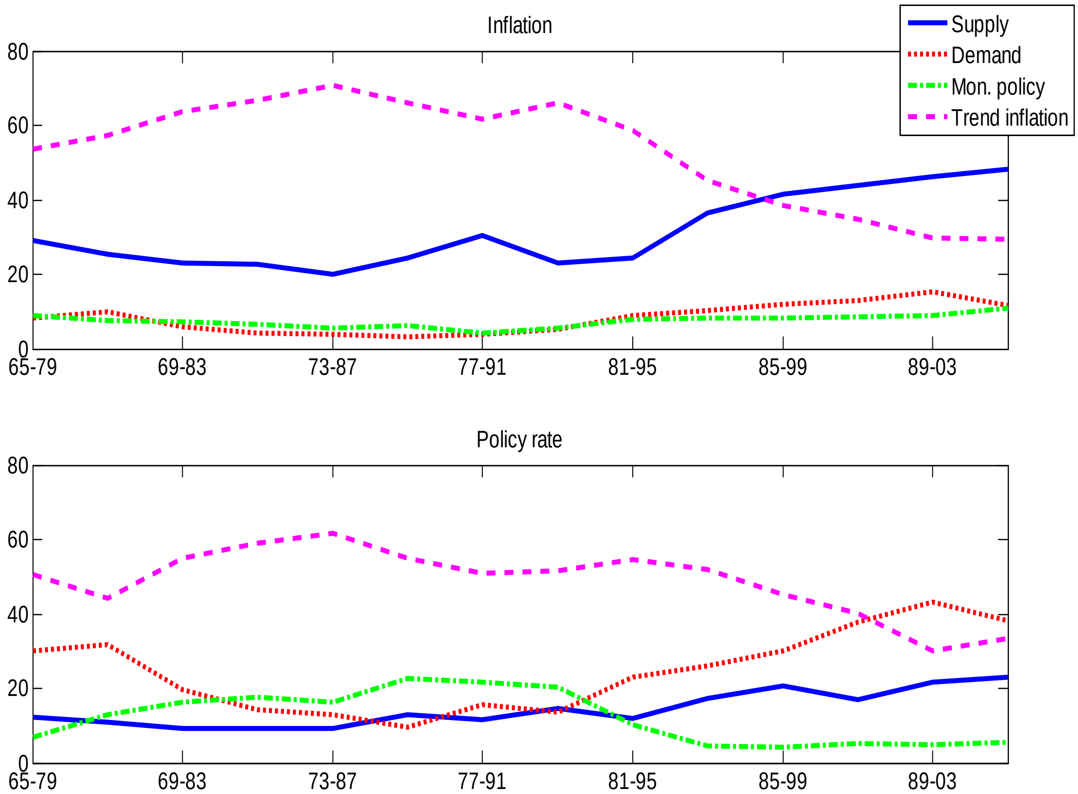

Figure 9: **Rolling-window variance decomposition implied by the Smets and Wouters (2007) model.** Model calibration: Window-specific posterior mode. "Supply" and "Demand" shocks defined as in Smets and Wouters (2007), i.e., supply shocks: TFP, price mark-up, and wage mark-up shocks; demand shocks: Investment-specific tech., Government spending, and risk-premium shocks. Version of the model with reaction to the output gap set to zero to ease the computation of the posterior mode (very similar results were obtained by relaxing this constraint).