Appendix of the paper "In Cholesky-VARs We Trust? An Empirical Investigation with U.S. Data"

1 CVAR estimated with actual U.S. data, 1954:III-2008:II

The evidence shown in the paper - Figures 1 and 4 - is clearly not in line with the one provided by a variety of authors as for the effects of a monetary policy shock over the entire post-WWII sample (Christiano, Eichenbaum, and Evans (1999), Christiano, Eichenbaum, and Evans (2005)). This Section shows that the discrepancy between our results, based on great moderation data, and those obtained with longer samples is mainly driven by the observations belonging to the 1970s. Figures A1 and A2 document our CVAR evidence obtained with the sample 1954:III-2008:II with two different trivariate VARs, one featuring the CBO measure of the output gap and the other one modeling quarterly output growth. Evidently, the reactions of inflation and output are significant and quite different with respect to the mild-to-muted ones shown in the paper. In particular, inflation reacts positively and significantly to a monetary policy tightening, so confirming the existence of a "price puzzle" as already documented by, among others, Boivin and Giannoni (2006) and Castelnuovo and Surico (2010). The reaction of output is in line with conventional wisdom, in that it signals a recession, whose timing and persistence is shown to depend on the business cycle indicator employed in each given VAR.

2 Bayesian estimation

To perform our Bayesian estimations we employed `DYNARE`, a set of algorithms developed by Michel Juillard and collaborators (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011)). `DYNARE` is freely available at the following URL: http://www.dynare.org/.

The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first $t \leq t_0 = 20,000$ draws. To do so, we computed the posterior mode by the "csminwel" algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix C_0 of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the "Adaptive Metropolis" (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient than the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to 'tune' the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix C_t of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, see Haario, Saksman, and Tamminen (2001).

We simulated two chains of 200,000 draws each, and discarded the first 90% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations.

3 Further results on the small-scale model

3.1 Predictive power of the estimated small-scale model

We checked the predictive power of the estimated small-scale model. Figure A3 contrasts the actual series employed in our empirical exercise with the DSGE model's one step-ahead predictions. As shown by the Figure, the model performs well along the one-step ahead forecasting dimension.

3.2 The role of the "timing discrepancy"

Are the distortions induced by the Cholesky-decomposition *quantitatively* relevant? Figure A4 displays the histograms of the distribution of the quarter-specific percentage deviations. The distributions are clearly shifted leftward with respect to the zero value, so indicating underestimation of the true effects of a monetary policy shock, or wrongly signed responses. The 90% sets suggest that these distortions are important also once sample uncertainty is accounted for. The median deviation reads -102% for inflation, and -97% for the output gap, i.e. the deviations from the true impulse responses are clearly sizeable.

Why do we get distorted impulse responses with our CVARs? The fundamental reason is the discrepancy in the *timing assumptions* entertained by the DSGE vs. CVAR models. While the first one allows for an *immediate* impact of the policy shock on inflation and output, the CVAR imposes a *delayed* reaction. To understand the consequences of this timing issue, let's stick to our small-scale model and consider the set of unique decision rules consistent with the rational expectation assumption and the structure of our small-scale DSGE model:[1]

$$\begin{bmatrix} \pi_t \\ y_t \\ R_t \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} \varepsilon_t^{\pi} \\ a_t \\ \varepsilon_t^{R} \end{bmatrix}, \mathbf{\Gamma} \equiv \begin{bmatrix} a_1 & f_1 & e_1 \\ a_2 & f_2 & e_2 \\ a_3 & f_3 & e_3 \end{bmatrix}, \boldsymbol{B} \equiv \begin{bmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{bmatrix} \tag{1}$$

where $\mathbf{\Gamma}$ and $\boldsymbol{B}$ collect convolutions of the structural parameters $\boldsymbol{\xi}$ of the DSGE model. Given that the third column of $\boldsymbol{B}$ does not display, in general, zeros, the monetary policy shock ε_t^R *immediately* affects *all* the variables of the system.

The small-scale model has the following VAR(2) representation:

$$\begin{bmatrix} \pi_t \\ y_t \\ R_t \end{bmatrix} = \boldsymbol{A}_1 \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{bmatrix} + \boldsymbol{A}_2 \begin{bmatrix} \pi_{t-2} \\ y_{t-2} \\ R_{t-2} \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} u_t^{\pi} \\ u_t^{a} \\ u_t^{R} \end{bmatrix} \tag{2}$$

where $\boldsymbol{A}_1 = \mathbf{\Gamma} + \boldsymbol{BFB}^{-1}$ and $\boldsymbol{A}_2 = -\boldsymbol{BFB}^{-1}\mathbf{\Gamma}$. The variance-covariance matrix of $\boldsymbol{Bu}$ is given by $\boldsymbol{B\Omega B}^T$, where $\boldsymbol{\Omega}$ is a diagonal matrix of full rank 3 with the variances of the shocks positioned on the main diagonal. For ease of exposition (and without loss of generality), we set $\boldsymbol{\Omega} = \boldsymbol{I}_3$.

[1] The theoretical part of this Section heavily relies on the derivations proposed by Carlstrom, Fuerst, and Paustian (2009).

Of course, when conducting an econometric exercise, the fundamental shocks $\boldsymbol{u}_t$ are not observable, and must be inferred. To do so, the econometrician can estimate a reduced form VAR(2)

$$\begin{bmatrix} \pi_t \\ y_t \\ R_t \end{bmatrix} = \boldsymbol{A}_1 \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ R_{t-1} \end{bmatrix} + \boldsymbol{A}_2 \begin{bmatrix} \pi_{t-2} \\ y_{t-2} \\ R_{t-2} \end{bmatrix} + \begin{bmatrix} \zeta_t^\pi \\ \zeta_t^a \\ \zeta_t^R \end{bmatrix},$$

where $\boldsymbol{\zeta}_t$ is a vector of *residuals* whose variance-covariance $VCV(\boldsymbol{\zeta}) = \boldsymbol{\Lambda}$ is a full (non diagonal) $[3x3]$ matrix.

To recover the unobserved structural monetary policy shock u_t^R, a researcher must impose some restrictions on the structure of the VAR, e.g. the simultaneous relationships among the variables included in the vector, the long-run impact of some economic shocks, or the sign of some conditional correlations. The most popular choice is to orthogonalize the residuals by imposing a Cholesky structure to the system, which assumes delayed effects of the 'monetary policy shock' on the variables located before the nominal interest rate in the vector $[\pi_t, y_t, R_t]^T$. This is done by computing the unique lower triangular matrix $\widetilde{\boldsymbol{B}}$ such that

$$\widetilde{\boldsymbol{B}}\boldsymbol{\varphi_t} = \boldsymbol{\zeta}, \text{ with } \widetilde{\boldsymbol{B}} = \begin{bmatrix} \widetilde{b}_1 & 0 & 0 \\ \widetilde{b}_2 & \widetilde{c}_2 & 0 \\ \widetilde{b}_3 & \widetilde{c}_3 & \widetilde{d}_3 \end{bmatrix}, \text{ and } \boldsymbol{\varphi_t} = \begin{bmatrix} \varphi_t^\pi \\ \varphi_t^a \\ \varphi_t^R \end{bmatrix}. \tag{3}$$

The Cholesky "shocks" $\boldsymbol{\varphi_t}$, which are orthogonal and are assumed to have unitary variance, are then identified by computing the elements of the matrix $\widetilde{\boldsymbol{B}}$ such that

$$\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T = \boldsymbol{\Lambda}.$$

This implies that the equivalence $\widetilde{\boldsymbol{B}}\widetilde{\boldsymbol{B}}^T = \boldsymbol{B}\boldsymbol{B}^T$ must hold. Solving the system, it is then possible to express the elements of $\widetilde{\boldsymbol{B}}$ in terms of the objects belonging to $\boldsymbol{B}$.

Given the restriction

$$\widetilde{\boldsymbol{B}}\boldsymbol{\varphi}_t = \boldsymbol{B}\boldsymbol{u}_t \tag{4}$$

imposed by eqs. (2) and (3), one may express the Cholesky-'shocks' $\boldsymbol{\varphi_t}$ in terms of the DSGE shocks $\boldsymbol{u}_t$ and the elements belonging to the matrix $\boldsymbol{B}$.

$$\boldsymbol{\varphi}_t = \Phi\boldsymbol{u}_t = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} u_t^\pi \\ u_t^a \\ u_t^R \end{bmatrix}, \tag{5}$$

where $\Phi \equiv \widetilde{\boldsymbol{B}}^{-1}\boldsymbol{B}$. Therefore, the mapping going from the true DSGE shocks to the CVAR monetary policy 'shock' reads

$$\varphi_t^R = \phi_{31}u_t^\pi + \phi_{32}u_t^a + \phi_{33}u_t^R. \tag{6}$$

The "shock" φ_t^R is, in fact, a misspecified representation of the true monetary policy shock u_t^R. The standard Cholesky identification scheme recovers the true policy shock only under the restrictions $\phi_{31} = \phi_{32} = 0$. These would occur under $d_1 = d_2 = 0$ in the monetary impulse vector $\boldsymbol{B}[:,3]$ in eq. (1), i.e. if the structural DSGE model would feature lags in the impact of the true monetary policy shock u_t^R on inflation and output. However, these restrictions are *not* consistent with the DSGE models employed by most researchers, the model we focus on in this paper included. The calibration conditional on our estimated posterior means implies the following values for the matrices characterizing the set of decision rules (1):

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0.08 & 0.03 & -0.27 \\ -0.03 & 0.25 & -0.62 \\ 0.03 & 0.02 & 0.68 \end{bmatrix}, and \boldsymbol{B} = \begin{bmatrix} 1.78 & -0.10 & -0.63 \\ -5.22 & -0.17 & -1.22 \\ 0.59 & -0.05 & 0.70 \end{bmatrix}.$$

Notably, $\boldsymbol{B}[1,3] = d_1 = -0.63$, and $\boldsymbol{B}[2,3] = d_2 = -1.22$. As a consequence, the Cholesky scheme misspecifies the monetary policy shock.

Figure A5 (top row) plots these densities. Interestingly, the cost-push shock u_t^π enters the reduced form CVAR monetary policy shock with a weight close to zero on average. Differently, the distribution of the weight ϕ_{32} assigned to the technology shock u_t^a is negative and 'significantly' different from zero, with a mean equal to -0.40. Also the density of the loading ϕ_{33} of the shock u_t^R suggests values different from zero, and displays a mean close to 6.5.

The variance decomposition analysis of the reduced form shock φ^R is depicted in Figure A5 (bottom row). The contribution of the true technology shock u_t^a is on average 31%, a figure stressing that the misspecification induced by the imposition of the Cholesky scheme is substantial. In theory, also the cost-push shock u_t^π could contribute to bias the reduced form monetary policy shock φ_t^R. In practice, however, its contribution is negligible, with a mean around 1%. The remaining volatility is due to the true monetary policy shock u_t^R.

These findings offer a rationale for the distorted CVAR responses we obtain with our MonteCarlo exercises. The stochastic element identified by the CVAR monetary policy 'shock' is in fact a convolution of the true technology shock u_t^a, which enters the reduced form φ_t^R with a *negative* weight, and the true monetary policy shock u_t^R, which enters it with a *positive* sign. A negative technology shock opens a positive output gap, which exerts a positive pressure on inflation and the policy rate. At the same time, a monetary policy shock (a policy tightening) triggers a positive reaction of the policy rate, and a negative reaction of inflation and the output gap. Then, the reduced form shock φ_t^R actually captures the *joint* effects of these two structural shocks, which basically offset each other as for inflation and output, leading to muted reactions like those depicted in Figures 3.

4 The Smets-Wouters (2007) model

The Smets and Wouters (2007) model is a Dynamic Stochastic General Equilibrium framework extremely popular in academic and institutional circles. The model features a number of shocks and frictions, which offer a quite rich representation of the economic environment and allow for a satisfactory in-sample fit of a set of macroeconomic data (Del Negro, Schorfheide, Smets, and Wouters (2007)). Moreover, Smets and Wouters (2007) show that this model is quite competitive when contrasted with Bayesian-VARs as for forecasting exercises, in particular for the elaboration of medium-term predictions.

The Smets and Wouters (2007) model features sticky nominal price and wage settings that allow for backward-looking inflation indexation; habit formation in consumption; investment adjustment costs; variable capital utilization and fixed costs in production. The stochastic dynamics is driven by seven structural shocks, namely a total factor productivity shock, two shocks affecting the intertemporal margin (risk premium shocks and investment-specific technology shocks), two shocks affecting the intratemporal margin (wage and price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks).

In a nutshell, the model features the following main ingredients. Households maximize a nonseparable utility function in consumption and labor over an infinite life horizon. Consumption appears in the utility function in quasi-difference form with respect to a time-varying external habit variable. Labor is differentiated by a union, so there is some monopoly power over wages, which results in explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. The utilization of the capital stock can be adjusted at increasing cost. Firms produce differentiated goods, decide on labor and capital inputs, and set prices conditional on the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that are not reoptimized are partially indexed to past inflation rates. Prices are therefore set in function of current and expected marginal costs, but are also determined by the past inflation rate. The marginal costs depend on wages and the rental rate of capital. Similarly, wages depend on past and expected future wages and inflation. The model features, in both goods and labor markets, an aggregator that allows for a time-varying demand elasticity depending on the relative price as in Kimball (1995). This is important because the introduction of real rigidity allows us to estimate a more reasonable degree of price and wage stickiness.

The log-linearized version of the DSGE model around its steady-state growth path reads as follows:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \quad (7)$$

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \varepsilon_t^b) \quad (8)$$

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i \quad (9)$$

$$q_t = q_1 E_t q_t + 1 + (1 - q_1) E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \varepsilon_t^b) \quad (10)$$

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a) \quad (11)$$

$$k_t^s = k_{t-1} + z_t \quad (12)$$

$$z_t = z_1 r_t^k \quad (13)$$

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i \quad (14)$$

$$\mu_t^p = \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t \quad (15)$$

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \quad (16)$$

$$r_t^k = -(k_t - l_t) + w_t \quad (17)$$

$$\mu_t^w = w_t - (\sigma_l l_t + (1 - \lambda/\gamma)^{-1} (c_t - \lambda/\gamma c_{t-1})) \quad (18)$$

$$w_t = w_1 w_{t-1} + w_2 (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \quad (19)$$

$$r_t = \rho r_{t-1} + (1 - \rho)(r_\pi + r_Y (y_t - y_t^p)) + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^R \quad (20)$$

$$\varepsilon_t^x = \rho_x \varepsilon_{t-1}^x + \eta_t^x, x = (b, i, a, R) \quad (21)$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \quad (22)$$

$$\varepsilon_t^z = \rho_x \varepsilon_{t-1}^z + \eta_t^z - \chi_z \eta_{t-1}^z, z = (p, w) \quad (23)$$

$$\eta_t^j \sim N(0, \sigma_j^2) \quad (24)$$

where:

$$c_y = 1 - g_y - i_y \tag{25}$$

and g_y and i_y are the steady-state exogenous spending-output ratio and investment-output ratio, with:

$$i_y = (\gamma - 1 + \delta)k_y \tag{26}$$

where γ is the steady-state growth rate, δ is the depreciation rate of capital, k_y is the steady-state capital-output ratio; $z_y = R_*^y k_y$ is the steady-state rental rate of capital. Notice that eq. (22), the one of the stochastic process of the government spending, allows for the productivity shock to affect it. This is so because exogenous spending, in this model, includes net exports, which may be affected by domestic productivity development.

As for the consumption Euler equation (8):

$$c_1 = \frac{\lambda}{\gamma}\left(1 + \frac{\lambda}{\gamma}\right) \tag{27}$$

$$c_2 = \frac{(\sigma_c - 1)\frac{W_*^h L_*}{C_*}}{\sigma_c(1 + \frac{\lambda}{\gamma})} \tag{28}$$

$$c_3 = \frac{1 - \frac{\lambda}{\gamma}}{\left(1 + \frac{\lambda}{\gamma}\right)\sigma_c} \tag{29}$$

Current consumption is a function of past and expected future consumption, of expected growth in hours worked, of the ex ante real interest rate, and of a disturbance term ε_t^b. Under the assumption of no habits ($\lambda = 0$) and that of log-utility in consumption ($\sigma_c = 1$), $c_1 = c_2 = 0$, then the standard purely forward looking consumption equation is obtained. The disturbance term ε_t^b represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive shock to this wedge increases the required return on assets held by the households. At the same time, it increases the cost of capital and it decreases the value of capital and investment (see below). This is basically a shock very similar to a net-worth shock. This disturbance is assumed to follow a standard AR(1) process.

The dynamics of investment is captured by the investment Euler equation (9), where:

$$i_1 = \frac{1}{1 + \beta\gamma^{1-\sigma_c}} \tag{30}$$

$$i_2 = \frac{1}{1 + \beta\gamma^{1-\sigma_c}\gamma^2\varphi} \tag{31}$$

where φ is the steady-state elasticity of the capital adjustment cost function, and β is the discount factor applied by households. Notice that capital adjustment costs are a function of the change in investment, rather than its level. This choice is made to introduce additional dynamics in the investment equation, which is useful to capture the hump-shaped response of investment to various shocks. In this equation, the stochastic disturbance ε_t^i represents a shock to the investment-specific technology process, and is assumed to follow a standard first-order autoregressive process.

The value-of-capital arbitrage equation (10) suggests that the current value of the capital stock q_t depends positively on its expected future value (with weight $q_1 = \beta\gamma^{-\sigma_c}(1-\delta)$), as well as the expected real rental rate on capital $E_t r_{t+1}^k$ and on the ex ante real interest rate and the risk premium disturbance.

Eq. (11) is the first one of the supply side block. It describes the aggregate production function, which maps output to capital (k_t^s) and labor services (l_t). The parameter α captures the share of capital in production, and the parameter ϕ_p is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

Eq. (12) suggest that the newly installed capital becomes effective with a one-period delay, hence current capital services in production are a function of capital installed in the previous period k_t and the degree of capital utilization z_t. As stressed by eq. (13), the degree of capital utilization is a positive function of the rental rate of capital, $z_t = z_1 r_t^k$, where $z_1 = (1-\psi)/\psi$ and ψ is a positive function of the elasticity of the capital utilization adjustment cost function normalized to belong to the [0,1] domain.

Eq. (14) describes the accumulation of installed capital k_t, featuring the convolutions:

$$k_1 = (1-\delta)/\gamma \tag{32}$$

$$k_2 = \left[1-\left(1-\frac{\delta}{\gamma}\right)\right]\left(1+\beta\gamma^{1-\sigma_c}\right)\gamma^2\varphi \tag{33}$$

Installed capital is a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance ε_t^i, which follows an autoregressive, stationary process.

Eq. (15) relates to the monopolistic competitive goods market. Cost minimization by firms implies that the price mark-up μ_t^p, defined as the difference between the average price and the nominal marginal cost or the negative of the real marginal cost, is equal to the difference between the marginal product of labor and the real wage w_t, with the marginal product of labor being itself a positive function of the capital-labor ratio and total factor productivity.

Profit maximization by price-setting firms gives rise to the New-Keynesian Phillips curve, i.e., eq. (16), with the convolutions being:

$$\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}, \tag{34}$$

$$\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}, \tag{35}$$

$$\pi_3 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{\xi_p\left[(\phi_p-1)\varepsilon_p+1\right]}. \tag{36}$$

Notice that, in maximizing their profits, firm have to face price stickiness à la Calvo (1983). Firms that cannot reoptimize in a given period index their prices to past inflation as in Smets and Wouters (2003). In equilibrium, inflation π_t depends positively on past and expected future inflation, negatively on the current price mark-up, and positively on a price mark-up disturbance ε_t^p. The price mark-up disturbance is assumed to follow an ARMA(1,1) process. The inclusion of the MA term is to grab high-frequency fluctuations in inflation. When the degree of price indexation $\iota_p = 0$, $\pi_1 = 0$ and eq. (16) collapses to the purely forward-looking, standard NKPC. The assumption that all prices are indexed to either lagged inflation or trend inflation ensures the verticality of the Phillips curve in the long run. The speed of adjustment to the desired mark-up depends, among others, on the degree of price stickiness ξ_p, the curvature of the Kimball goods market aggregator ε_p, and the steady-state mark up, which in equilibrium is itself related to the share of fixed costs in production $(\phi_p - 1)$ via a zero-profit condition. In particular, when all prices are flexible ($\xi_p = 0$) and the price mark-up shock is zero at all times, eq. (16) reduces to the familiar condition that the price mark-up is constant, or equivalently that there are no fluctuations in the wedge between the marginal product of labor and the real wage. Cost minimization by firms also implies that the rental rate of capital is negatively related to the capital-labor ratio and positively to the real wage (both with unitary elasticity) (see eq. (17)).

Similarly, in the monopolistically competitive labor market, the wage mark-up will be equal to the difference between the real wage and the marginal rate of substitution between working and consuming, an equivalence captured by eq. (18), where σ is the elasticity of labor supply with respect to the real wage and λ is the habit parameter in consumption. Eq. (19) shows that real wages adjust only gradually to the desired wage mark-up due to nominal wage stickiness and partial indexation, the convolutions related to this equation being:

$$
\begin{aligned}
w_1 &= \frac{1}{1+\beta\gamma^{1-\sigma_c}} && (37)\\
w_2 &= \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}} && (38)\\
w_3 &= \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}} && (39)\\
w_4 &= \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}\frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_w)(1-\xi_w)}{\xi_w\left[(\phi_w-1)\varepsilon_w+1\right]} && (40)
\end{aligned}
$$

Notice that if wages are perfectly flexible ($\xi_w = 0$), the real wage is a constant mark-up over the marginal rate os substitution between consumption and leisure. When wage indexation is zero ($\iota_w = 0$), real wages do not depend on lagged inflation. Notice that, symmetrically with respect to the pricing scheme analyzed earlier, also the wage-mark up disturbance follows an ARMA(1,1) process.

The model is closed by eq. (20), which is a flexible Taylor rule postulating a systematic reaction by policymakers to current values of inflation, the output gap, and output growth. In particular, one of the objects policymakers react to is the output gap, defined as a difference between actual and potential output (in logs). Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks. Then, policymakers engineer movements in the short-run policy rate r_t, movements which happen gradually given the presence of interest rate smoothing ρ. Stochastic departures from the Taylor rate, i.e. the rate that would realize in absence of any policy rate shocks, are triggered by a stochastic AR(1) process.

Finally, eqs. (21)-(24) define the stochastic processes of the model, which features, as already pointed out, seven shocks (total factor productivity, investment specific technology, risk premium, exogenous spending, price mark-up, wage mark-up, and monetary policy).

Notice that the model features a deterministic growth rate driven by labor-augmenting technological progress, so that the data do not need to be detrended before estimation.

5 Further results with Smets and Wouters (2007)

Figure A6 plots the results of our MonteCarlo exercise conditional on an estimated version of the Smets and Wouters (2007) featuring no reaction to the output gap by the Federal Reserve, which focuses on inflation and output growth only. The similarity between this Figure and Figure 4 in the text is striking, i.e., our results are not affected by this perturbation in the policy rule.

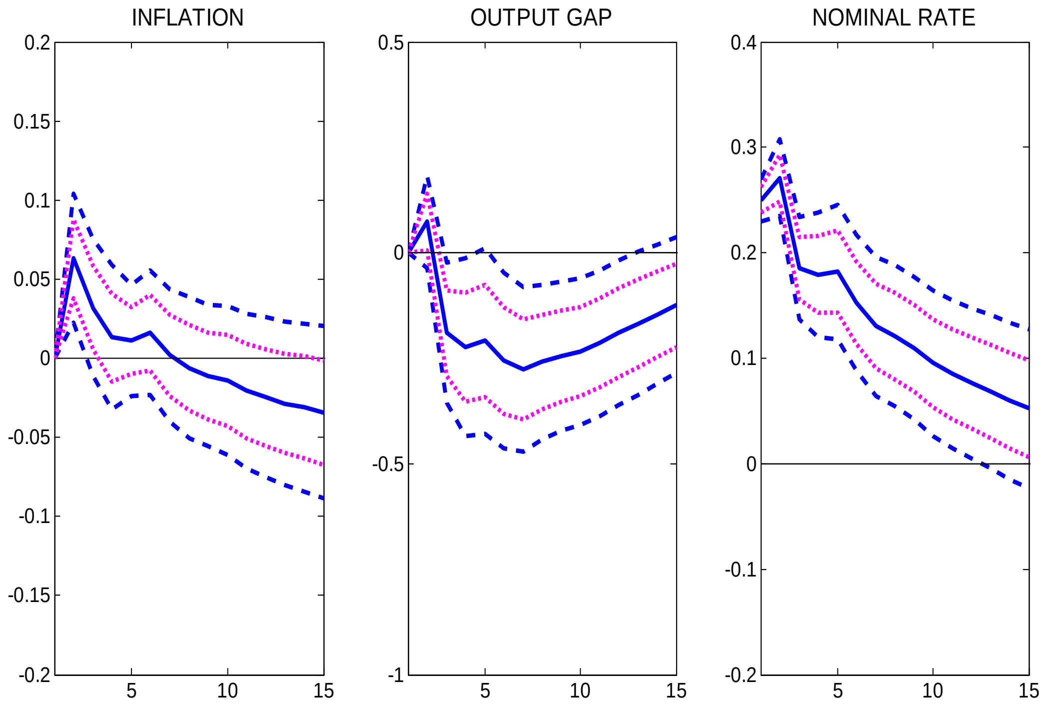

Figure A1: **CVAR impulse response functions to a monetary policy shock, 1954:III-2008:II - model with CBO output gap**. Variables: Quarterly GDP inflation, CBO output gap, quarterly federal funds rate - source: FREDII. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output growth, federal funds rate). Solid blue line: Mean response; Dashed blue lines: [5th,95th] percentiles; Magenta dotted lines: [16th,84th] percentiles (bootstrapped, 500 repetitions). VAR estimated with a constant, a linear trend, and four lags.

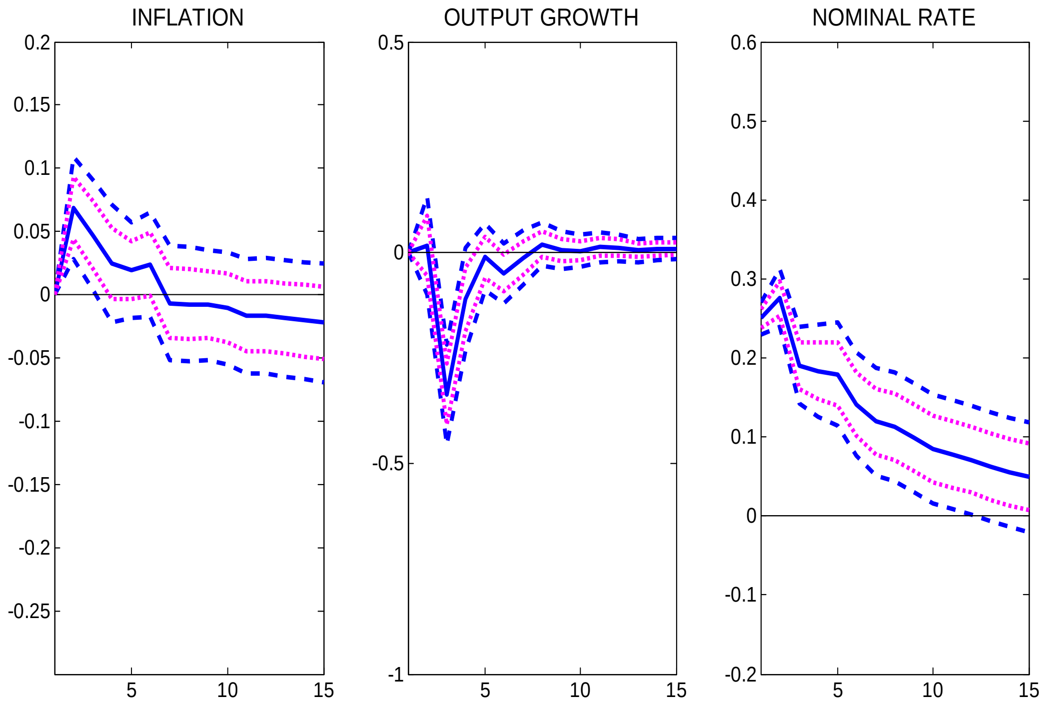

Figure A2: **CVAR impulse response functions to a monetary policy shock, 1954:III-2008:II - model with output growth**. Variables: Quarterly GDP inflation, quarterly output growth, quarterly federal funds rate - source: FREDII. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output growth, federal funds rate). Solid blue line: Mean response; Dashed blue lines: [5th,95th] percentiles; Magenta dotted lines: [16th,84th] percentiles (bootstrapped, 500 repetitions). VAR estimated with a constant, a linear trend, and four lags.

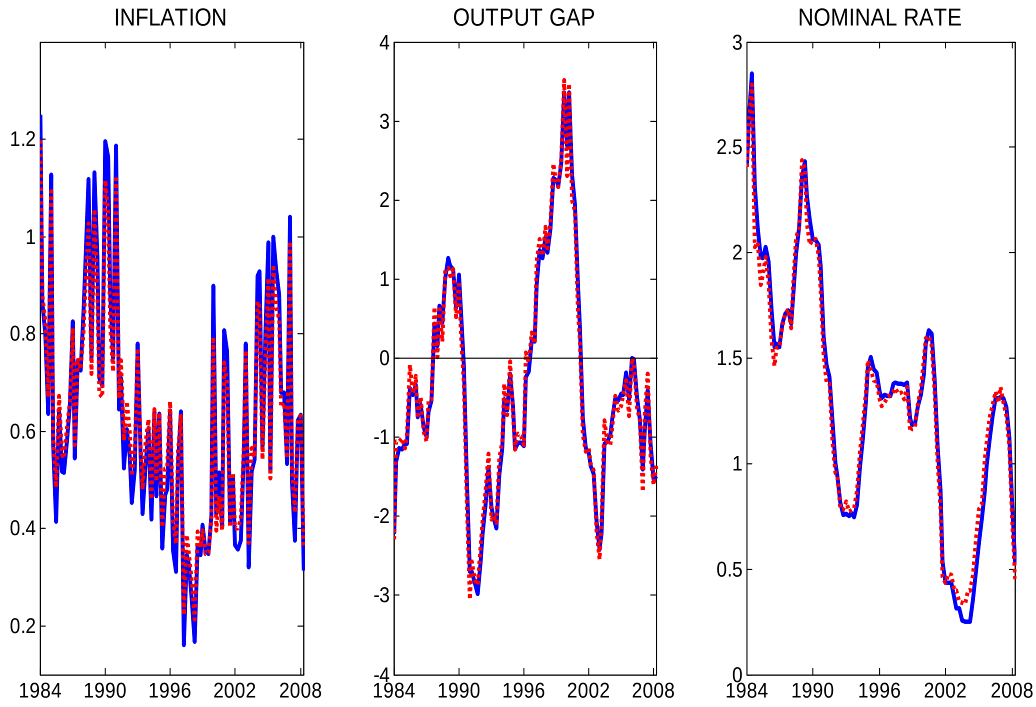

Figure A3: **Actual series vs. DSGE's one-step ahead forecasts.** Solid blue line: Actual series; Dotted red lines: DSGE's one-step-ahead predictions.

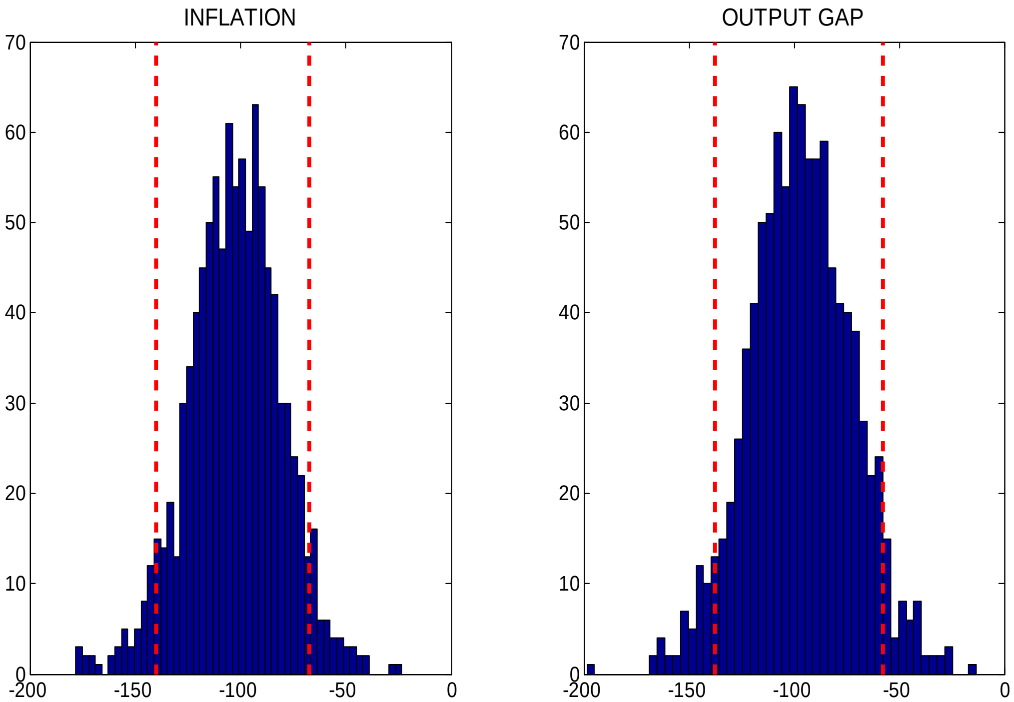

Figure A4: **Impulse response functions: Percentage deviations**. Percentage deviations of the CVAR responses with respect to the DSGE (true) responses - one quarter after the shock. Red dotted lines: [5th,95th] percentiles. Computation of the densities based on 5,000 draws of the structural parameters of the DSGE model.

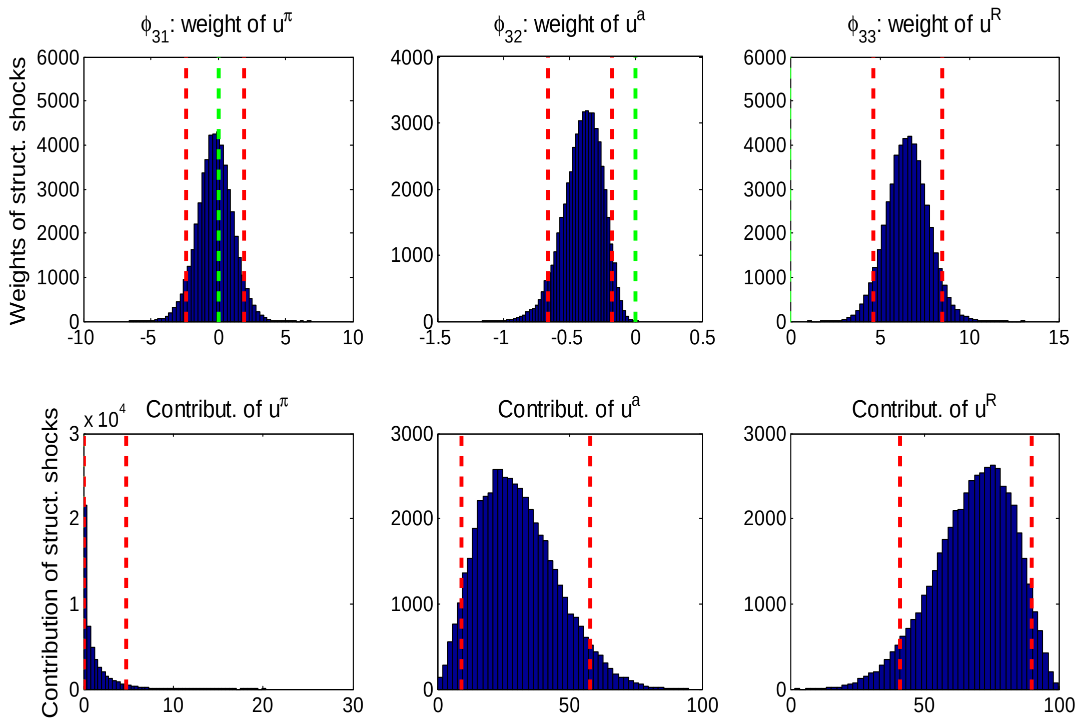

Figure A5: **CVAR monetary policy shock: Weights and contributions of the DSGE's shocks.** Distribution computed over 5,000 stochastic simulations.

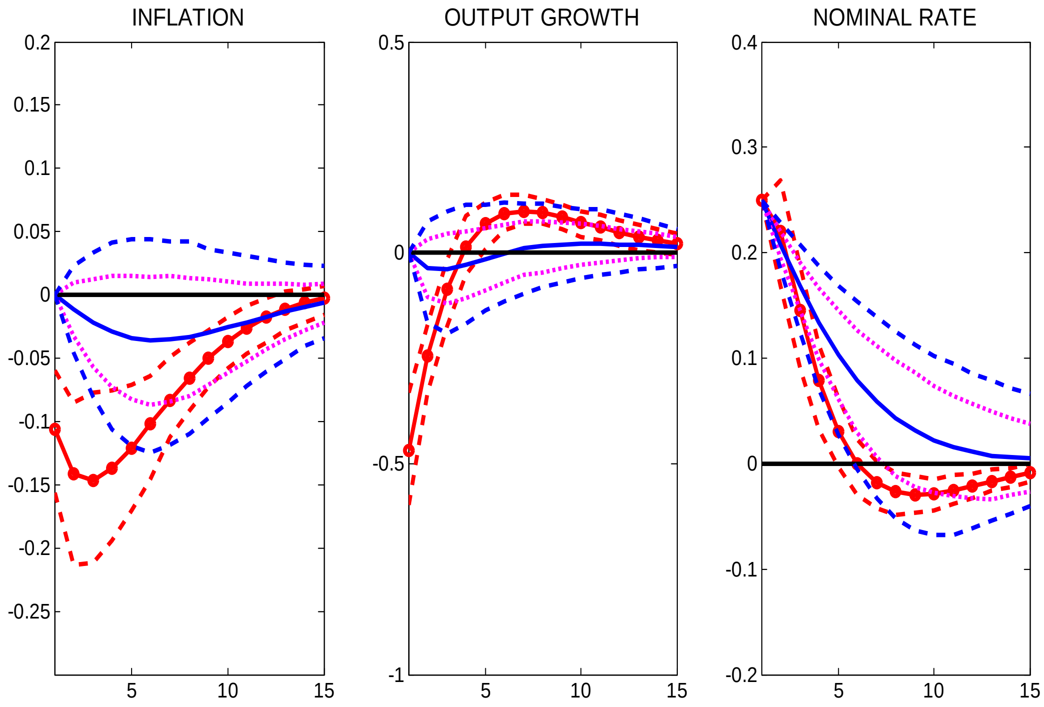

Figure A6: **DSGE à la Smets and Wouters (2007) vs. CVAR impulse response functions to a monetary policy shock - rule featuring no output gap.** Circled red lines: DSGE Bayesian mean impulse responses; Dashed red lines: 90% credible sets. Solid blue line: CVAR mean impulse responses; Dashed blue lines: [5th,95th] percentiles; Magenta dotted lines: [16th,84th] percentiles. Moments computed the impulse response function distributions simulated by drawing 5,000 realizations of the vector of parameters of the DSGE model, which is also used to generate the pseudo-data to feed the CVARs. Identification of the monetary policy shock via Cholesky decomposition (lower triangular matrix, ordering: inflation, output growth, nominal rate). VAR estimated with a number of lags determined (per each given VAR) by the Schwarz criterion.

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